PML and high accuracy BIE solver for wave scattering by a locally defected periodic surface

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Kylin lectures in Numerical analysis,
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Overview

Contents

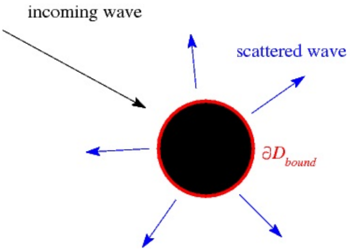
Classic scattering problems

u^{inc}: plane or point source wave.

$$(\partial_{x_1}^2 + \partial_{x_2}^2) u^{sc} + k^2 u^{sc} = 0, \quad \text{in} \quad \mathbb{R}^2/\bar{D},$$
$$u^{sc} = -u^{inc}, \quad \text{on} \quad \partial D.$$

Boundary condition at ∞:
Sommerfeld radiation condition
(Sommerfeld 1912):

$$\lim_{r\to\infty} \sqrt{r} \left(\frac{\partial u^{sc}}{\partial r} - iku^{sc} \right) = 0.$$



Borrowed from the internet.

Perfectly matched layer

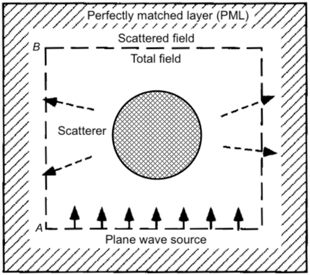
Mathematically, the PML (Berenger JCP 1994) introduces complexified coordinate transformations

$$\tilde{x}_j = x_j + \mathrm{i}S \int_0^{x_2} \sigma_j(t)dt, j = 1, 2, \quad (1)$$

and truncates the domain by enforcing the following boundary condition

$$u^{\mathrm{sc}}(\tilde{x}_1, \tilde{x}_2) = 0,$$

on the PML boundary.



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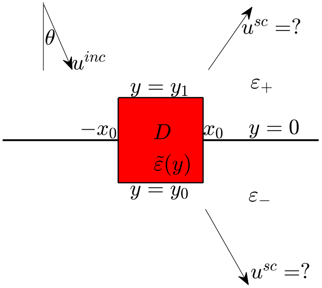
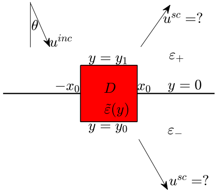
Layerd-medium scattering problem

Γ: A local perturbation of $x_2 = 0$.



u^{sc} no longer satisfies Sommerfeld radiation condition.

Sommerfeld Radiation Condition



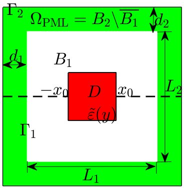
u^{sc}_{ref}: the scattered wave due to plane wave u^{inc} incident on $x_2=0$, i.e., the unperturbed case when $D=\emptyset$. Computable!
u^{sc} satisfies:
$u^{og} := u^{sc} - u^{sc}_{ref}$ satisfies Sommerfeld radiation condition in $\mathbb{R}^2/\Gamma$, i.e.,

$$\partial_r u^{og} - ik_{\pm} u^{og} = O(r^{-1/2}), \quad r\to\infty, \quad \mathrm{in}\mathbb{R}^2_{\pm}.$$

This condition still gaurantees uniqueness and existence of u^{sc} (see Monk 2003, Chen & Zheng 2012, Bao, Hu and Yin 2018).
Advantage in computation: Perfectly Matched Layer (Berenger JCP 1994) absorbs u^{og}.

Perfectly matched layer

Since u^{og} satisfies Sommerfeld radiation condition, we could use PML to absorb it.



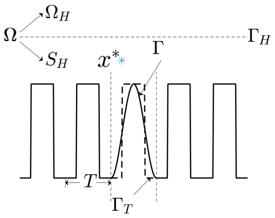
PML:

$$\tilde{x}_j = x_j + \mathrm{i}S \int_0^{x_j} \sigma_j(t)dt, j = 1, 2. \tag{2}$$

We enforce

$$\tilde{u}^{og}(x_1, x_2) =: u^{og}(\tilde{x}_1, \tilde{x}_2) = 0, \quad \text{on} \quad \Gamma_2.$$

Our Problem



Governing equations:

$$\Delta u^{\text{tot}} + k^2 u^{\text{tot}} = 0, \quad \text{on} \quad \Omega, \tag{3}$$

$$u^{\text{tot}} = 0, \quad \text{on} \quad \Gamma, \tag{4}$$

Geometrical condition:

$$\text{(GC1)}: \quad (x_1, x_2) \in \Omega \Rightarrow (x_1, x_2 + a) \in \Omega, \quad \forall a \geq 0,$$

(GC2): some (and hence any) period of Γ_T contains a line segment,

Motivation

- Does u^{tot} or any related function satisfies the SRC?
- How to truncate the computational domain by using the PML?
- How to efficiently and accurately compute u^{tot}?

Radiation condition

Two types of incidences:

(i) a plane wave $u^{\mathrm{inc}}(x) = e^{ik(\cos\theta x_1 - \sin\theta x_2)}$ for the incident angle $\theta \in (0, \pi)$;

(ii) a cylindrical wave $u^{\mathrm{inc}}(x; x^*) = G(x; x^*) = \frac{\mathrm{i}}{4} H_0^{(1)}(k|x - x^*|)$ excited by a source at $x^* = (x_1^*, x_2^*) \in \Omega$.

Sommerfeld radiation condition[1]:

(i). For the plane-wave incidence, $u^{\mathrm{og}} := u^{\mathrm{tot}} - u^{\mathrm{tot}}_{\mathrm{ref}}$, where $u^{\mathrm{tot}}_{\mathrm{ref}}$ is the reference scattered field for the unperturbed scattering curve $\Gamma = \Gamma_T$, satisfies the following half-plane Sommerfeld radiation condition (hSRC): for some sufficiently large $R > 0$ and any $\rho < 0$,

$$\lim_{r\to\infty} \sup_{\alpha\in[0,\pi]} \sqrt{r}\,|\partial_r u^{\mathrm{og}}(x) - \mathrm{i}k u^{\mathrm{og}}(x)| = 0, \ \sup_{r\geq R} r^{1/2}|u^{\mathrm{og}}(x)| < \infty,$$
$$\text{and } u^{\mathrm{og}} \in H^1_\rho(S_H^R), \tag{5}$$

where $x = (r\cos\alpha, H + r\sin\alpha)$, $S_H^R = S_H \cap \{x : |x_1| > R\}$, and $H^1_\rho(\cdot) = (1 + x_1^2)^{-\rho/2} H^1(\cdot)$ denotes a weighted Sobolev space.

(ii). For the cylindrical incidence, the total field $u^{\mathrm{og}} := u^{\mathrm{tot}}$ itself satisfies the hSRC (5) in Ω_H. Thus, the scattered field u^{sc} satisfies (5) as well since u^{inc} satisfies (5).

[1] Hu, **L.**, Rathsfeld, SIAP, 2021

Related works

- Well-posedness theory:
  - Chandler-Wilde and Monk, SIMA, 2005
  - Chandler-Wilde and Elschner, SIMA,2010
  - Hu, **L.** and Rathsfeld, SIAP, 2021
- PML convergence theory:
  - Chen and Wu, SINUM, 2003
  - Chandler-Wilde and Monk, ANM, 2009
  - Zhou and Wu, JSC, 2018
- Boundary conditions for defected periodic structures:
  - Joly, Li and Fliss, CICP, 2006
  - Yuan and Lu, JLT, 2007
  - Ehrhardt, Han and Zheng, CICP, 2009
  - Sun and Zheng, JOSAA, 2009
  - Hu and Lu, IEEEPTL, 2009
  - Lechleiter and Zhang, SISC, 2017

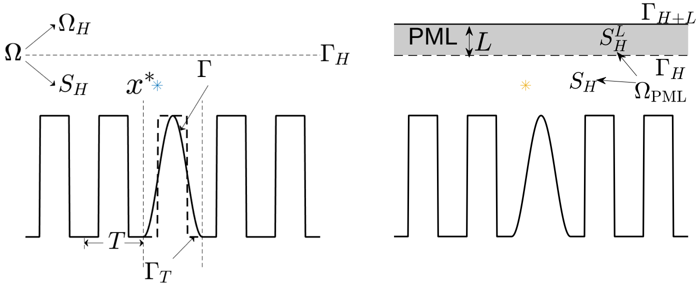
PML

Based on the radiation condition, the PML only truncates x_2, and introduces a complexified coordinate transformation

$$\tilde{x}_2 = x_2 + \mathrm{i}S \int_0^{x_2} \sigma(t)dt, \tag{6}$$

where

$$\sigma(x_2) = \begin{cases} \frac{2f_2^m}{f_1^m + f_2^m}, & x_2 \in [H, H + L/2] \\ 2, & x_2 \geq H + L/2, m \neq 0 \\ 1, & x_2 \geq H + L/2, m = 0 \\ 0, & x_2 \leq 0. \end{cases} \tag{7}$$



Well-posedness and convergence results

Let $\tilde{u}^{\rm og}(x;x^*) := u^{\rm og}(\tilde{x};x^*)$. It satisfies the PML-truncated problem:

$$\begin{aligned}\nabla\cdot(\mathrm{A}\nabla\tilde{u}^{\rm og})+k^2\alpha(x_2)\tilde{u}^{\rm og} &= -\,\delta(x-x^*), \quad \text{on } \Omega_{\rm PML},\\ \tilde{u}^{\rm og} &= 0, \quad \text{on} \quad \Gamma,\\ \tilde{u}^{\rm og} &= 0, \quad \text{on} \quad \Gamma_{H+L}=\{x:x_2=H+L\},\end{aligned}$$

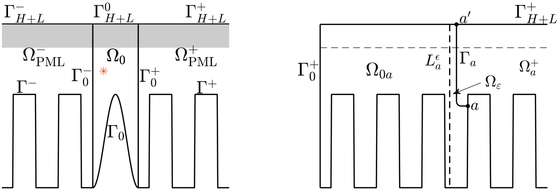
where $\alpha(x_2)=1+\mathrm{i}S\sigma(x_2)$ and $\mathrm{A}=\mathrm{Diag}\{\alpha,\alpha^{-1}\}$.

Theorem (Yu et al., 2021)

Provided that $\tilde{S}L$ is sufficiently large where $\tilde{S}=\frac{S}{L}\int_H^{H+L}\sigma(t)dt$, the PML-truncated problem admits a unique solution $\tilde{u}^{\rm og}(x;x^*)=\tilde{u}_r^{\rm og}(x;x^*)+\chi(x;x^*)\tilde{u}^{\rm inc}(x;x^*)$ with $\tilde{u}_r^{\rm og}\in H_0^1(\Omega_{\rm PML})=\{\phi\in H^1(\Omega_{\rm PML}):\phi|_{\Gamma\cup\Gamma_{H+L}}=0\}$ for any $x^*\in\Omega_{\rm PML}$.

Contents

Semiwaveguide problems



$$(P^\pm):\quad \begin{cases} \nabla\cdot(\mathrm{A}\nabla\tilde{u}) + k^2\alpha\tilde{u} = 0, \quad \text{on} \quad \Omega^\pm_{\mathrm{PML}} := \Omega_{\mathrm{PML}} \cap \left\{x : \pm x_1 > \frac{T}{2}\right\}, \\ \tilde{u} = 0, \quad \text{on} \quad \Gamma^\pm := \Gamma \cap \left\{x : \pm x_1 > \frac{T}{2}\right\}, \\ \tilde{u} = 0, \quad \text{on} \quad \Gamma^\pm_{L+H} := \Gamma_{L+H} \cap \left\{x : \pm x_1 > \frac{T}{2}\right\}, \\ \partial_{\nu_c}\tilde{u} = g^\pm, \quad \text{on} \quad \Gamma^\pm_0 := \Omega_{\mathrm{PML}} \cap \left\{x : x_1 = \pm\frac{T}{2}\right\}, \end{cases}$$

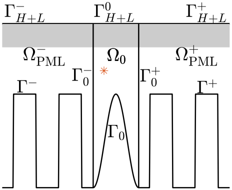
Well-posedness

Theorem (Yu et al., 2021)

Under the geometrical conditions (GC1) and (GC2), provided that $\tilde{S}L$ is sufficiently large, the semi-waveguide problem ($P^{\pm}$) has a unique solution $\tilde{u} \in H^1(\Omega^{\pm}_{\mathrm{PML}})$ such that $||\tilde{u}||_{H^1(\Omega^{\pm}_{\mathrm{PML}})} \leq C||g^{\pm}||_{H^{-1/2}(\Gamma^{\pm}_0)}$ for any $g^{\pm} \in H^{-1/2}(\Gamma^{\pm}_0)$, respectively, where C is independent of $g^{\pm}$.

Exact lateral boundary conditions

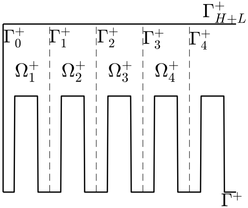
The well-posedness theorem implies that we can define two vertical Neumann-to-Dirichlet (vNtD) operators $\mathcal{N}^{\pm} : H^{-1/2}(\Gamma_0^{\pm}) \to \widetilde{H^{1/2}}(\Gamma_0^{\pm})$ satisfying $\tilde{u}^{\mathrm{og}}|_{\Gamma_0^{\pm}} = \mathcal{N}^{\pm}\partial_{\nu_c}\tilde{u}^{\mathrm{og}}|_{\Gamma_0^{\pm}}$.



$$\begin{cases} \nabla \cdot (A\nabla \tilde{u}^{\mathrm{og}}) + k^2 \alpha \tilde{u}^{\mathrm{og}} = -\delta(x - x^*), & \text{on } \Omega_0, \\ \tilde{u}^{\mathrm{og}} = 0, & \text{on } \Gamma_0 = \Gamma \cap \{x : |x_1| < T/2\}, \\ \tilde{u}^{\mathrm{og}} = 0, & \text{on } \Gamma_{H+L}^0 = \Gamma_{H+L} \cap \{x : |x_1| < T/2\}, \\ \tilde{u}^{\mathrm{og}} = \mathcal{N}^{\pm}\partial_{\nu_c}\tilde{u}^{\mathrm{og}}, & \text{on } \Gamma_0^{\pm}. \end{cases}$$

Marching operators

Marching operators: $\mathcal{R}_p^{\pm} : H^{-1/2}(\Gamma_0^{\pm}) \to H^{-1/2}(\Gamma_1^{\pm})$ satisfying $\partial_{\nu_c^{\pm}} \tilde{u}^{\mathrm{og}}|_{\Gamma_1^{\pm}} = \mathcal{R}_p^{\pm} \partial_{\nu_c^{\pm}} \tilde{u}^{\mathrm{og}}|_{\Gamma_0^{\pm}}$.



Lemma (Yu et al., 2021)

Under the conditions that (GC2) holds and $\tilde{S}L$ is sufficiently large, we can choose $\Gamma_0^{\pm}$ intersecting Γ at a smooth point such that $\mathcal{R}_p^{\pm}$ are compact operators and

$$\partial_{\nu_c^{\pm}} \tilde{u}^{\mathrm{og}}|_{\Gamma_{j+1}^{\pm}} = \mathcal{R}_p^{\pm} \partial_{\nu_c^{\pm}} \tilde{u}^{\mathrm{og}}|_{\Gamma_j^{\pm}}, \tag{8}$$

holds for any $j \geq 0$. Furthermore,

$$\rho(\mathcal{R}_p^{\pm}) < 1, \tag{9}$$

where ρ denotes the spectral radius.

Exponentially decaying property

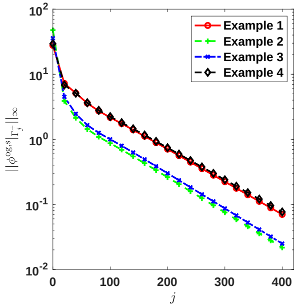
Corollary (Yu et al., 2021)

Under the conditions that (GC2) holds and $\tilde{S}L$ is sufficiently large,

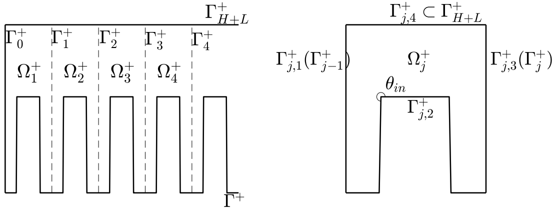
$$||\tilde{u}^{\rm og}(\cdot;x^*)||_{H^1(\Omega_j^{\pm,N_0})} \leq C||(\mathcal{R}_p^{\pm})^{N_0}||^{j-1}||\tilde{g}^{\rm inc}||_{L^2(\Omega_{\rm PML})}, \qquad (10)$$

where C is independent of $j \geq 0$. In other words, the PML truncated solution $\tilde{u}^{\rm og}(x;x^*)$ decays exponentially fast to 0 in the strip as $|x_1| \to \infty$ for any $x^* \in \Omega_{\rm PML}$.

As a consequence, this reveals that the PML truncation cannot realize an exponential convergence to the true solution since the true solution, as indicated by Chandler-Wilde and Monk 2005, behaves as $\mathcal{O}(x_1^{-3/2})$ as $x_1 \to \infty$.



Riccati equations



Consider the following boundary value problem for a generic field $\tilde{u}$:

$$\begin{cases} \nabla \cdot (\mathsf{A}\nabla\tilde{u}) + k^2\alpha\tilde{u} = 0, & \text{on } \Omega_j^+, \\ \tilde{u} = 0, & \text{on } \Gamma_{j,2} \cup \Gamma_{j,4}, \\ \partial_{\nu_c}\tilde{u} = g_i, & \text{on } \Gamma_i^+, i = j-1, j, \end{cases}$$

for $g_i \in H^{-1/2}(\Gamma_i^+), i = j-1, j$.

Theorem (Yu et al., 2021)

Provided that $kT/\pi \notin \mathcal{E} := \{i'/2^{j'} | j' \in \mathbb{N}, i' \in \mathbb{N}^*\}$, and L is sufficiently large, the above problem is well-posed. The well-posedness even holds with Ω_j^+ replaced by the interior domain of 2^l consecutive cells, say $\cup_{j=1}^{2^l}\overline{\Omega_j^+}$, for any number $l \geq 0$.

We can define a bounded Neumann-to-Dirichlet operator $\mathcal{N}^{(0)} : H^{-1/2}(\Gamma_{j-1}^{+}) \times H^{-1/2}(\Gamma_{j}^{+}) \to \widetilde{H^{1/2}}(\Gamma_{j-1}^{+}) \times \widetilde{H^{1/2}}(\Gamma_{j}^{+})$ such that

$$\begin{bmatrix} \tilde{u}|_{\Gamma_{j-1}^{+}} \\ \tilde{u}|_{\Gamma_{j}^{+}} \end{bmatrix} = \mathcal{N}^{(0)} \begin{bmatrix} \partial_{\nu_c^-} \tilde{u}|_{\Gamma_{j-1}^{+}} \\ \partial_{\nu_c^+} \tilde{u}|_{\Gamma_{j}^{+}} \end{bmatrix}, \tag{11}$$

for all $j \geq 1$. Let

$$\mathcal{N}^{(0)} = \begin{bmatrix} \mathcal{N}_{00}^{(0)} & \mathcal{N}_{01}^{(0)} \\ \mathcal{N}_{10}^{(0)} & \mathcal{N}_{11}^{(0)} \end{bmatrix}.$$

Thus,

$$\begin{aligned} \mathcal{N}_{10}^{(0)} \partial_{\nu_c^-} \tilde{u}^{\mathrm{og}}|_{\Gamma_0^+} - \mathcal{N}_{11}^{(0)} \mathcal{R}_p^+ \partial_{\nu_c^-} \tilde{u}^{\mathrm{og}}|_{\Gamma_0^+} &= \tilde{u}^{\mathrm{og}}|_{\Gamma_1^+} \\ &= \mathcal{N}_{00}^{(0)} \mathcal{R}_p^+ \partial_{\nu_c^-} \tilde{u}^{\mathrm{og}}|_{\Gamma_0^+} - \mathcal{N}_{01}^{(0)} (\mathcal{R}_p^+)^2 \partial_{\nu_c^-} \tilde{u}^{\mathrm{og}}|_{\Gamma_0^+}. \end{aligned}$$

We get two Riccati equations

$$\begin{aligned} \mathcal{N}_{10}^{(0)} + [\mathcal{N}_{11}^{(0)} + \mathcal{N}_{00}^{(0)}] \mathcal{R}_p^+ + \mathcal{N}_{01}^{(0)} (\mathcal{R}_p^+)^2 &= 0, \\ \mathcal{N}_{01}^{(0)} + [\mathcal{N}_{11}^{(0)} + \mathcal{N}_{00}^{(0)}] \mathcal{R}_p^- + \mathcal{N}_{10}^{(0)} (\mathcal{R}_p^-)^2 &= 0, \end{aligned}$$

for $\mathcal{R}_p^{\pm}$. Then the lateral NtD operators are given by

$$\begin{aligned} \mathcal{N}^+ &= \mathcal{N}_{00}^{(0)} - \mathcal{N}_{01}^{(0)} \mathcal{R}_p^+, \\ \mathcal{N}^- &= \mathcal{N}_{11}^{(0)} - \mathcal{N}_{10}^{(0)} \mathcal{R}_p^-. \end{aligned}$$

Recursive doubling procedure

We first study the NtD operator

$$\mathcal{N}^{(l)} = \begin{bmatrix} \mathcal{N}_{00}^{(l)} & \mathcal{N}_{01}^{(l)} \\ \mathcal{N}_{10}^{(l)} & \mathcal{N}_{11}^{(l)} \end{bmatrix} \tag{12}$$

on the boundary of $\cup_{j=1}^{2^l} \overline{\Omega_j^+}$ for $l \geq 1$.

$$\mathcal{N}_{00}^{(l)} = \mathcal{N}_{00}^{(l-1)} - \mathcal{N}_{01}^{(l-1)} \mathcal{A}_{l-1}, \quad \mathcal{N}_{01}^{(l)} = \mathcal{N}_{01}^{(l-1)} \mathcal{B}_{l-1}, \tag{13}$$

$$\mathcal{N}_{10}^{(l)} = \mathcal{N}_{10}^{(l-1)} \mathcal{A}_{l-1}, \quad \mathcal{N}_{11}^{(l)} = \mathcal{N}_{11}^{(l-1)} - \mathcal{N}_{10}^{(l-1)} \mathcal{B}_{l-1}. \tag{14}$$

We have

$$\mathcal{N}_{10}^{(l)} + [\mathcal{N}_{11}^{(l)} + \mathcal{N}_{00}^{(l)}](\mathcal{R}_p^+)^{2^l} + \mathcal{N}_{01}^{(l)} (\mathcal{R}_p^+)^{2^{(l+1)}} = 0, \tag{15}$$

$$\mathcal{N}^+ = \mathcal{N}_{00}^{(l)} - \mathcal{N}_{01}^{(l)} (\mathcal{R}_p^+)^{2^l}. \tag{16}$$

Since $||(\mathcal{R}_p^+)^{N_0}|| < 1$, the third term in (15) is expected to be exponentially small for $l \gg \log_2 N_0$.

$$(\mathcal{R}_p^+)^{2^l} \approx -[\mathcal{N}_{11}^{(l)} + \mathcal{N}_{00}^{(l)}]^{-1} \mathcal{N}_{10}^{(l)}, \tag{17}$$

$$\mathcal{N}^+ \approx \mathcal{N}_{00}^{(l)} + \mathcal{N}_{01}^{(l)} [\mathcal{N}_{11}^{(l)} + \mathcal{N}_{00}^{(l)}]^{-1} \mathcal{N}_{10}^{(l)}, \tag{18}$$

and we get $\mathcal{R}_p^+$ iteratively from

$$(\mathcal{R}_p^+)^{2^j} = -[\mathcal{N}_{11}^{(j)} + \mathcal{N}_{00}^{(j)}]^{-1} \left[\mathcal{N}_{10}^{(j)} - \mathcal{N}_{01}^{(j)} (\mathcal{R}_p^+)^{2^{j+1}} \right], \quad j = l-1, \cdots, 0.$$

Performance

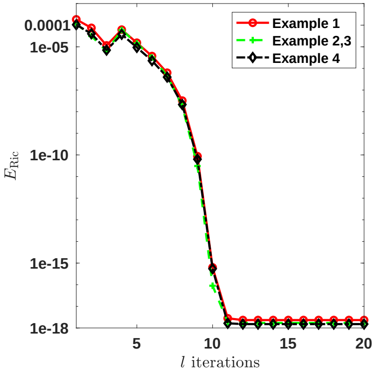


Figure: Convergence history against the number of iterations l.

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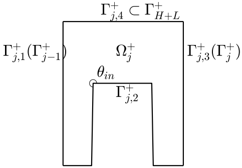
Neumann-to-Dirichlet operator



For any function $\tilde{u}$ satisfying

$$\nabla \cdot (A \nabla \tilde{u}) + k^2 \alpha \tilde{u} = 0, \quad \text{on} \quad \Omega_1^+. \tag{19}$$

We have[1] $\tilde{u} = (\mathcal{K} - \mathcal{K}_0[1])^{-1} \mathcal{S} \partial_{\nu_c} \tilde{u}$ on $\partial\Omega_1^+$, where

$$\mathcal{S}[\phi](x) = 2 \int_{\partial\Omega_1^+} \tilde{G}(x,y)\phi(y)ds(y),$$

$$\mathcal{K}[\phi](x) = 2\text{p.v.} \int_{\partial\Omega_1^+} \partial_{\nu_c} \tilde{G}(x,y)\phi(y)ds(y),$$

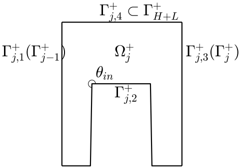
$$\mathcal{K}_0[\phi](x) = 2\text{p.v.} \int_{\partial\Omega_1^+} \partial_{\nu_c} \tilde{G}_0(x,y)\phi(y)ds(y),$$

and

$$\tilde{G}(x,y) = \frac{i}{4} H_0^{(1)}(k\sqrt{(\tilde{x}_1 - \tilde{y}_1)^2 + (\tilde{x}_2 - \tilde{y}_2)^2}).$$

[1] **L.**, Lu, Qian, SIAP, 2017

Approximating $\mathcal{N}^{(0)}$



$$\begin{bmatrix} \mathsf{u}_{1,1} \\ \mathsf{u}_{1,2} \\ \mathsf{u}_{1,3} \\ \mathsf{u}_{1,4} \end{bmatrix} = \mathsf{N}_u \begin{bmatrix} \phi^{\mathrm{s}}_{1,1} \\ \phi^{\mathrm{s}}_{1,2} \\ \phi^{\mathrm{s}}_{1,3} \\ \phi^{\mathrm{s}}_{1,4} \end{bmatrix}, \tag{20}$$

By $\tilde{u}|_{\Gamma_{1,2}\cup\Gamma_{1,4}} = 0$, we get

$$\begin{bmatrix} \mathsf{u}_{1,1} \\ \mathsf{u}_{1,3} \end{bmatrix} = \mathsf{N}^{(0)} \begin{bmatrix} \phi^{\mathrm{s}}_{1,1} \\ \phi^{\mathrm{s}}_{1,3} \end{bmatrix},$$

Contents

Example 1. Flat surface

Source point $y = (0, 1.5)$. Line segment $(-0.5, 0.5) \times \{x_2 = 0\}$ is assumed to be the perturbed part. $n = 1.03$ and $k_0 = 2\pi$.

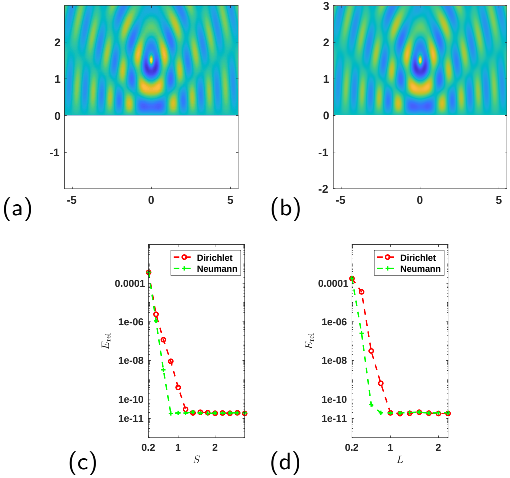


Figure: (a) exact solution; (b) numerical solution. Convergence history of relative error $E_{\rm rel}$ versus: (c) PML absorbing constant S; (d) Thickness of the PML L, for both Dirichlet and Neumann conditions on Γ_{H+L}.

Example 2. A sine curve

Cylindrical wave:

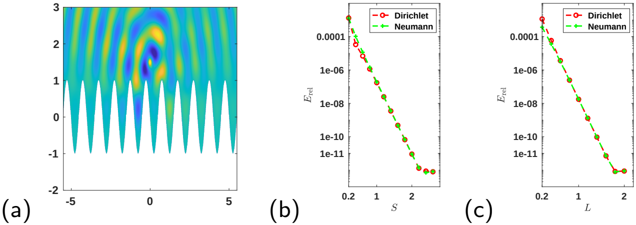


Figure: (a) Numerical solution of real part of the total wave field u in $[-5.5, 5.5]\times[-2.0, 3.0]$ excited by the point source $y=(0, 1.5)$. Convergence history of relative error $E_{\rm rel}$ versus: (b) PML absorbing constant S for fixed PML Thickness $L=2$, (c) PML Thickness L for fixed PML absorbing constant $S=2.8$; vertical axes are logarithmically scaled.

Example 2. A sine curve

Plane wave:

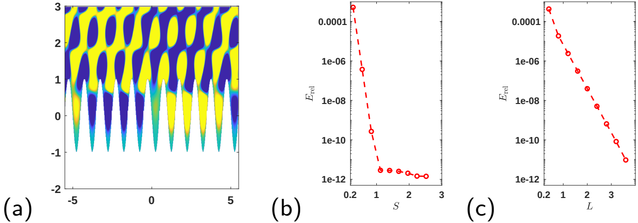


Figure: (a) Numerical solution of real part of the total wave field u in $[-5.5, 5.5] \times [-2.0, 3.0]$ excited by a plane incident wave of angle $\theta = \frac{\pi}{3}$. Convergence history of relative error $E_{\rm rel}$ versus: (b) PML absorbing constant S for fixed PML Thickness $L = 4$, (c) PML Thickness L for fixed PML absorbing constant $S = 2.8$; vertical axes are logarithmically scaled.

Example 3. A locally perturbed sine curve

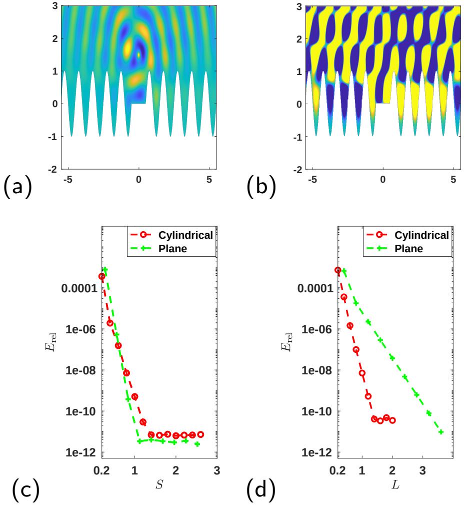


Figure: (a) a cylindrical wave by source $y = (0, 1.5)$; (b) a plane wave of incident angle $\theta = \frac{\pi}{3}$. Convergence history of relative error $E_{\rm rel}$ versus: (c) PML Thickness L for fixed PML absorbing constant $S = 2.8$ for both incidences; (d) PML absorbing constant S for fixed PML Thickness $L = 2.2$ (4.0) for cylindrical (plane-wave) incidence.

Example 4. A locally perturbed binary grating

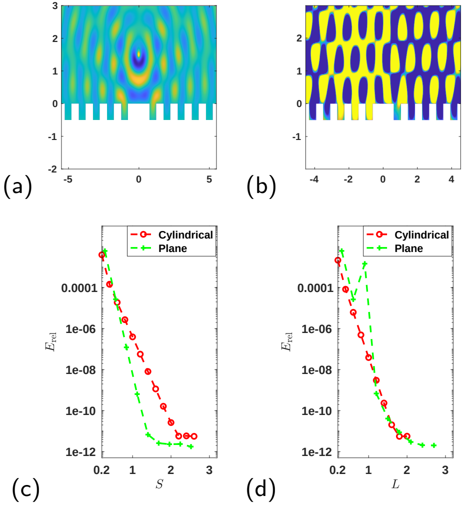


Figure: (a) a cylindrical wave by source $y=(0,1.5)$; (b) a plane wave of incident angle $\theta = \frac{\pi}{6}$. Convergence history of relative error $E_{\rm rel}$ versus: (c) PML Thickness L for fixed PML absorbing constant $S=2.8$ for both incidences; (d) PML absorbing constant S for fixed PML Thickness $L=2.2$ (3.0) for cylindrical (plane-wave) incidence.

Contents

Conclusions and Future works

Conclusions:

- A high-accuracy PML-BIE method has been developed for wave scatteing in locally perturbed periodic structures;
- Exact lateral boundary conditions were established to truncate the unbounded trip onto a bounded domain;
- Exponential convergence has been observed in a compact subset of the physical domain.

Future works:

- Extend the current work to study locally defected periodic structures of stratified media.
- Rigorously justify that the PML solution converges exponentially to the true solution in any compact subset of the strip.

Thanks for your attention!