Evolution Equations in Mathematical Physics: Dispersive and Hyperbolic

Tianjin University, Tianjin

2021. 09. 25—2021. 09. 26 (China)

2021. 09. 24—2021. 09. 26 (US)

Workshop Schedule

Time & Date (China time)	Saturday (September 25)	Sunday (September 26)
9:00-9:50	Wilhelm Schlag	Xiaoyi Zhang
9:50-10:40	Chongchun Zeng	Jason Murphy
10:40-11:30		Baoping Liu
21:00-21:50	Pierre Raphaël	Jinqiao Duan
21:50-22:40	Tadahiro Oh	Xiaohua Yao
22:40-23:30	Zoe Wyatt	Jia Shen

Zoom link（China Morning section）:

https://rutgers.zoom.us/j/99135402673?pwd=UnFuZlpLSHJRNXFuMXJvQXVkZkdXQT09;

ID：991 3540 2673； Password：795615

Zoom link（China Evening section）:

https://rutgers.zoom.us/j/91760759743?pwd=TFRVN2hUbTlrRzBQcnF1WHptZHFPQT09;

ID：917 6075 9743； Password：713442

Invited Talks

The Onsager-Machlup Action Functionals for Stochastic Partial Differential Equations

Jinqiao Duan (段金桥)

Illinois Institute of Technology, USA

Abstract. The Onsager-Machlup action functional provides statistical distribution information for a stochastic dynamical system. In particular, it quantifies how solution paths or trajectories stay in a given set (i.e, a ball or a tube) for a stochastic partial differential equation (dispersive or dissipative). It is equivalent to the Feynman path integral. The minimizer of the Onsager-Machlup action functional, in an appropriate function space, is the most probable transition path between metastable states. Therefore, these action functionals are used to investigate the most probable transition paths for stochastic dynamical systems arising in non-equilibrium systems, by calculus of variations.

Another way to see its significance: The Onsager-Machlup action functional is an analog of the Lagrangian action functional in classical mechanics.

The speaker will present a derivation of the Onsager-Machlup action functional for a lass of stochastic partial differential equations with (non-Gaussian) Levy motions as well as (Gaussian) Brownian motion. This is

achieved by applying the Girsanov transformation for probability measures and then by a path representation. This enables the investigation of the most probable transition path for infinite dimensional stochastic dynamical systems modeled by stochastic partial differential equations, by minimizing the Onsager-Machlup action functional.

This is joint work with Jianyu Hu.

Wellposedness for the KdV hierarchy

Baoping Liu (刘保平)

Peking University, China

Abstract. The KdV hierarchy is a hierarchy of integrable equations generalizing the KdV equation. Using the modified Muria transform, we first relate it to the Gardner hierarchy, and by exploiting the idea of approximate flow, we show that the whole hierarchy is wellposed for initial data in H^{-1}. This is based on joint work with H. Koch and F. Klaus.

Sharp scattering results for the 3d cubic NLS

Jason Murphy

Missouri University of Science and Technology, USA

Abstract. I will discuss several sharp scattering results for three-dimensional cubic nonlinear Schrödinger equations, including both the 'free' equation and the NLS with an external potential. After reviewing proofs of scattering below the mass/energy ground state threshold, I will discuss some recent work on scattering at the threshold for NLS with repulsive potentials. The talk will discuss joint works with B. Dodson; R. Killip, M. Visan, and J. Zheng; and C. Miao and J. Zheng.

Stochastic quantization of the Φ^3_3-model

Tadahiro Oh

The University of Edinburgh, United Kingdom

Abstract. We study the construction of the Φ^3_3-measure and complete the program on the (non-)construction of the focusing Gibbs measures, initiated by Lebowitz, Rose, and Speer (1988). This problem turns out to be critical, exhibiting a phase transition: normalizability in the weakly nonlinear regime and non-convergence of the truncated Φ^3_3-measures in the strongly nonlinear regime.

We also go over the dynamical problem for the canonical stochastic quantization of the Φ^3_3-measure, namely, the three-dimensional stochastic damped nonlinear wave equation with a quadratic nonlinearity forced by an additive space-time white noise (= the hyperbolic Φ^3_3-model). We first discuss briefly the paracontrolled approach for local well-posedness. In the globalization part, we introduce a new, conceptually simple and straightforward approach, where we directly work with the (truncated) Gibbs measure, using the variational formula and ideas from theory of optimal transport.

This is a joint work with Mamoru Okamoto (Osaka University) and Leonardo Tolomeo (University of Bonn).

Blow up for defocusing NLS

Pierre Raphaël

Cambridge University, United Kingdom

Abstract. I will explain the construction of blow up solutions for the defocusing NLS and will review its connection to compressible models in fluid dynamics.

Asymptotic stability for the Sine-Gordon kink under odd perturbations

Wilhelm Schlag

Yale University, USA

Abstract. We will describe the recent asymptotic analysis with Jonas Luehrmann of the Sine-Gordon evolution of odd data near the kink. We do not rely on the complete integrability of the problem in a direct way, in particular we do not use the inverse scattering transform.

Local and global solutions for the nonlinear Schrödinger equation in Fourier-Lebesgue spaces

Jia Shen (申佳)

Tianjin University, China

Abstract. In this talk, I will show a recent result about the well-posedness and long time behavior of nonlinear Schrödinger equation (NLS) in Fourier-Lebesgue spaces $\mathcal{F}L^p$. This work was finished joint with Professors Avy Soffer and Yifei Wu. Our main purpose is to study NLS with rough data that can possess infinite energy or mass.

First, we proved the local well-posedness of NLS with algebraic nonlinear terms in $\mathcal{F}L^1$. For more general nonlinear terms, we got the local results for energy sub-critical NLS in $\mathcal{F}L^p$ for some $p > 0$. As for the energy critical or super-critical NLS, the local theory was also obtained by adding some derivative assumptions. Especially for the energy critical case, only epsilon-order derivative is required.

The second main part is about some large data global well-posedness and scattering results for defocusing NLS. For the energy sub-critical and mass super-critical NLS, we proved the global well-posedness and scattering with initial data u_0 in $\mathcal{F}L^1$ and xu_0 in L^2. Particularly in this case, we do not impose any condition on the derivative of initial data. Moreover, we also established the global well-posedness and scattering for $3D$, defocusing, energy critical NLS with the initial data u_0 satisfying

$< D >^s u_0$ in $L^2 \cap \mathcal{F}L^1$ with some $s < 1$, where the long time behavior of the solution in the Fourier-Lebesgue space is also obtained.

Coupled wave and Klein-Gordon equations in two and three spatial dimensions

Zoe Wyatt

University of Cambridge, United Kingdom

Abstract. Semilinear wave equations in three spatial dimensions with wave--wave nonlinearities exhibit interesting and well-studied phenomena: from John's famous blow-up examples, to the null condition of Christodoulou and Klainerman, and more recently to the weak null condition of Lindblad and Rodnianski. The study of coupled semilinear wave and Klein-Gordon equations is less well-developed, and interesting problems occur across the possible spectrum of wave--wave, wave--KG and KG--KG interactions. In this talk I will discuss some recent results, in collaboration with Shijie Dong (Fudan), on such mixed systems. This includes a recent proof of small data global-existence and sharp asymptotics for a Dirac--Klein-Gordon system in two spatial dimensions.

Kato smoothing and Strichartz estimates for fractional and higher order operators with Hardy potentials

Xiaohua Yao (尧小华)

Central China Normal University, China

Abstract. Let $0 < \sigma < \frac{n}{2}$ and $H = (-\Delta)^{\sigma} + a|x|^{-2\sigma}$ be Schrödinger type operators on R^n with a sharp coupling constant $a < -C_{\sigma,n}$ ($C_{\sigma,n}$ is the best constant of Hardy's inequality of order σ). In the present talk, we will address that sharp global estimates for the resolvent and the solution to the time-dependent Schrödinger equation associated with H. In the case of the subcritical coupling constant $a > -C_{\sigma,n}$, we first prove the uniform resolvent estimates of Kato--Yajima type for all $0 < \sigma < \frac{n}{2}$, which turn out to be equivalent to Kato smoothing estimates for the Cauchy problem. We then establish Strichartz estimates for $\sigma > \frac{1}{2}$ and uniform Sobolev estimates of Kenig--Ruiz--Sogge type for $\sigma \geq \frac{n}{n+1}$. In the critical coupling constant case $a = -C_{\sigma,n}$, we show that the same results as in the subcritical case still hold for functions orthogonal to radial functions. This is a joint-work with Haruya Mizutani.

Small amplitude generalized breathers for nonlinear Klein-Gordon equations

Chongchun Zeng (曾崇纯)

Georgia Institute of Technology, USA

Abstract. Breathers are temporally periodic and spatially localized solutions of evolutionary PDEs. They are known to exist for integrable PDEs such as the sine-Gordon equation, but are believed to be rare for general nonlinear PDEs. When the spatial dimension is equal to one, exchanging the roles of time and space variables (in the so-called spatial dynamics framework), breathers can be interpreted as homoclinic solutions to steady solutions and thus arising from the intersections of the stable and unstable manifolds of the steady states. In this talk, we shall study small breathers of the nonlinear Klein-Gordon equation generated in an unfolding bifurcation as a pair of eigenvalues collide at the original when a parameter (temporal frequency) varies. Due to the presence of the oscillatory modes, generally the finite dimensional stable and unstable manifolds do not intersect in the infinite dimensional phase space, but with an exponentially small splitting (relative to the amplitude of the breather) in this singular perturbation problem of multiple time scales. This splitting leads to the transversal intersection of the center-stable and center-unstable manifolds which produces small amplitude generalized breathers with exponentially small tails. Due to the exponential small splitting, classical

perturbative techniques cannot be applied. We will explain how to obtain an asymptotic formula for the distance between the stable and unstable manifold of the steady solutions. This is a joint work with O. Gomide, M. Guardia, and T. Seara.

Dynamics of threshold solutions for focusing nonlinear Schrödinger equation with inverse square potential

Xiaoyi Zhang (张晓轶)

University of Iowa, USA

Abstract. In this talk, I will discuss our recent work on focusing Schrödinger equation with inverse square potential with the emphasis on the energy critical problem. We prove the existence and uniqueness of the stable/unstable manifold of the ground state and classify the solutions on the energy surface of the ground state. This is a joint work with Y. Kai and C. Zeng.