Kylin lectures in Numerical analysis #1: PDE Models

Tianjin University (Tianjin) &
SUSTech International Center for Mathematics (Shenzhen)

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Organizing Committee

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Workshop Schedule

Time & Date (Beijing time)	**Saturday (Dec. 04)**
15:00-15:50	Christian Lubich
15:50-16:40	Weizhu Bao
16:40-17:20	Liwei Xu
17:20-18:00	Wangtao Lu

Invited Talks

Multiscale methods and analysis for the Dirac equation in the nonrelativistic regime

Weizhu Bao

National University of Singapore, Singapore

Abstract. In this talk, I will review our recent works on numerical methods and analysis for solving the Dirac equation in the nonrelativistic regime, involving a small dimensionless parameter which is inversely proportional to the speed of light. In this regime, the solution is highly oscillating in time and the energy becomes unbounded and indefinite, which bring significant difficulty in analysis and heavy burden in numerical computation. We begin with four frequently used finite difference time domain (FDTD) methods and the time splitting Fourier pseudospectral (TSFP) method and obtain their rigorous error estimates in the nonrelativistic regime by paying particularly attention to how error bounds depend explicitly on mesh size and time step as well as the small parameter. Then we consider a numerical method by using spectral method for spatial derivatives combined with an exponential wave integrator (EWI) in the Gautschi-type for temporal derivatives to discretize the Dirac equation. Rigorous error estimates show that the EWI spectral method has much

better temporal resolution than the FDTD methods for the Dirac equation in the nonrelativistic regime. Based on a multiscale expansion of the solution, we present a multiscale time integrator Fourier pseudospectral (MTI-FP) method for the Dirac equation and establish its error bound which uniformly accurate in term of the small dimensionless parameter. Numerical results demonstrate that our error estimates are sharp and optimal. Finally, these methods and results are then extended to the nonlinear Dirac equation in the nonrelativistic regime and to the long-time dynamics of the Dirac equation with small electromagnetic potentials and the nonlinear Dirac equation with weak nonlinearity. This is a joint work with Yongyong Cai, Yue Feng, Xiaowei Jia, Qinglin Tang and Jia Yin.

PML and high-accuracy boundary integral equation solver for wave scattering by a locally defected periodic surface

Wangtao Lu

Zhejiang University, China

Abstract. In this talk, we shall study the perfectly-matched-layer (PML) method for wave scattering in a half space of homogeneous medium bounded by a two-dimensional, perfectly conducting, and locally defected periodic surface, and develops a high-accuracy boundary-integral-equation (BIE) solver. Along the vertical direction, we place a PML to truncate the unbounded domain onto a strip and prove that the PML solution converges to the true solution in the physical subregion of the strip with an error bounded by the reciprocal PML thickness. Laterally, we divide the unbounded strip into three regions: a region containing the defect and two semi-waveguide regions, separated by two vertical line segments. In both semi-waveguides, we prove the well-posedness of an associated scattering problem so as to well define a Neumann-to-Dirichlet (NtD) operator on the associated vertical segment. The two NtD operators, serving as exact lateral boundary conditions, reformulate the unbounded strip problem as a boundary value problem over the defected region. Due to the periodicity of the semi-waveguides, both NtD operators turn out to be closely related to a Neumann-marching operator, governed by a nonlinear Riccati equation.

It is proved that the Neumann-marching operators are contracting, so that the PML solution decays exponentially fast along both lateral directions. The consequences culminate in two opposite aspects. Negatively, the PML solution cannot converge exponentially to the true solution in the whole physical region of the strip. Positively, from a numerical perspective, the Riccati equations can now be efficiently solved by a recursive doubling procedure and a high-accuracy PML-based BIE method so that the boundary value problem on the defected region can be solved efficiently and accurately. Numerical experiments demonstrate that the PML solution converges exponentially fast to the true solution in any compact subdomain of the strip.

Time-dependent electromagnetic scattering from thin layers

Christian Lubich

Universität Tübingen, Germany

Abstract. The scattering of electromagnetic waves from obstacles with wave-material interaction in thin layers on the surface is described by generalized impedance boundary conditions, which provide effective approximate models. In particular, this includes a thin coating around a perfect conductor and the skin effect of a highly conducting material.

The approach taken in this work is to derive, analyse and discretize a system of time-dependent boundary integral equations that determines the tangential traces of the scattered electric and magnetic fields. In a second step the fields are evaluated in the exterior domain by a representation formula, which uses the time-dependent potential operators of Maxwell's equations.

A key role in the well-posedness of the time-dependent boundary integral equations and the stability of the numerical discretization is taken by the coercivity of the Calderón operator for the time-harmonic Maxwell's equations with frequencies in a complex half-plane. This entails the coercivity of the full boundary operator which includes the impedance operator.

The system of time-dependent boundary integral equations is

discretized with Runge--Kutta based convolution quadrature in time and Raviart--Thomas boundary elements in space. The full discretization is proved to be stable and convergent, with explicitly given rates in the case of sufficient regularity. The theoretical results are illustrated by numerical experiments.

Some results on boundary integral equation methods and their applications in numerics

Liwei Xu

University of Electronic Science and Technology, China

Abstract. Firstly, we discuss two results on integral equation methods associated to the solution of scattering wave equations. One is the new regularization formulation of the hypersingular boundary integral operators resulting from several elastic wave equations, and another is the well-posedness result of the approximated reduced boundary value problems corresponding to the original scattering transmission problems. Secondly, we present two results on applying integral equation methods to solve Laplace equations. One is on the change of order of integral operators so that some preconditioner of domain discretization methods could be applied to improve the efficiency of surface discretization methods, and another is a new coupling technique, i.e. the Dirichlet-to-Dirichlet or the Dirichlet-to-Neumann mapping defined on two different artificial boundaries, which could preserve the accuracy of the coupling scheme even as the mesh size tends to zero. The application of these theoretical results in numerics will be presented.