杰曼诺夫数学中心
SUSTech International Center for Mathematics

Kylin Lecture in analysis: Dispersive equations

Tianjin University (Tianjin) &
SUSTech International Center for Mathematics (Shenzhen)

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Organizing Committee

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ZOOM Info

ZOOM Link

Meeting ID: 986 9447 5021

Passcode: 1017

Workshop Schedule

Time & Date (Beijing time)	Sunday (Oct. 17)
10:00-10:50	Kenji Nakanishi (Plenary talk)
10:50-11:30	Zihua Guo
11:30-12:10	Shijie Dong
14:00-14:50	Zhifei Zhang (Plenary talk)
14:50-15:30	Nobu Kishimoto
15:30-16:10	Dongbing Zha
16:10-16:50	Jia Shen

Invited Talks

Some progress on nonlinear wave (and Klein-Gordon) equations

Shijie Dong (董世杰)

Fudan University, China

Abstract. I will discuss some recent progress on nonlinear wave equations and nonlinear wave-Klein-Gordon equations in two space dimensions, which is obtained via the hyperboloidal foliation method. This includes the new results on quasilinear wave equations (joint with Prof. LeFloch and Prof. Lei), on Klein-Gordon-Zakharov equations, and on Dirac-Klein-Gordon equations (joint with Wyatt).

Hardy-Sobolev space and its application in PDE

Zihua Guo (郭紫华)

Monash University, Australia

Abstract. Hardy space is of fundamental importance in analysis. It appears naturally in many occasions as a replacement of the Lebesgue space L^1. The Hardy space shares the same scaling as L^1, but behaves better in boundedness of many operators, especially for critical problems, due to the subtle structure (e.g. cancellation). We will talk about two applications of the Hardy-Sobolev space (function and its derivatives are in Hardy space). One is the local well-posedness of the Euler equation in the critical Hardy Sobolev space (Joint with Kuijie Li). The other one is the decay of the solutions for Schrödinger equation. We will also give an alternative proof of the boundedness of the Schrödinger propagator in Hardy space (Joint with Chunyan Huang, Liang Song).

Well-posedness for the periodic kinetic derivative nonlinear Schrödinger equation

Nobu Kishimoto

Kyoto University, Japan

Abstract. We consider a certain nonlinear Schrödinger equation (NLS) with nonlocal derivative nonlinearity, which is called the kinetic derivative NLS (KDNLS). The Cauchy problem for the standard derivative NLS has been proved to be locally well-posed in low regularity by the gauge transform or by its complete integrability, but these techniques cannot be directly adapted to KDNLS. On the other hand, KDNLS has dissipative nature, and especially in the periodic setting, a first-order parabolic term arises from the resonant nonlinear interactions. Taking advantage of this parabolic structure, we prove small-data local and global well-posedness results for periodic KDNLS in low-regularity Sobolev spaces. This is a joint work with Yoshio Tsutsumi (Kyoto University).

Global dynamics around 2-solitons for the nonlinear damped Klein-Gordon equation

Kenji Nakanishi

Kyoto University, Japan

Abstract. This is joint work with Kenjiro Ishizuka (Kyoto). We study global behavior of solutions for the Klein-Gordon equation with the damping term and the focusing power nonlinearity on the Euclidean space. Recently, Cote, Martel and Yuan proved the soliton resolution conjecture for this equation completely in the one-dimensional case: every global solution in the energy space is asymptotic to superposition of solitons. Then a natural question is which initial data evolve into each of the asymptotic forms. To answer this question, we consider global behavior of solutions starting near a superposition of two ground states with the opposite signs. The main result is a complete classification of such solutions into 5 types of global behavior. It contains two manifolds of solutions asymptotic to the ground states. They are joined at their boundary by the manifold of solutions asymptotic to superposition of two solitons, which was constructed by Cote, Martel, Yuan and Zhao. The connected union of the three manifolds separates the rest into the open set of global decaying solutions and that of blow-up. I will also talk about the difficulty in the same question around 3-solitons, namely soliton merger.

Almost sure well-posedness and scattering for $3D$ defocusing cubic NLS

Jia Shen (申佳)

Tianjin University, China

Abstract. I will show a recent work about the random data problem for $3D$ defocusing cubic NLS under the Wiener randomization, which is finished joint with Professors Avy Soffer and Yifei Wu. We improved the previous local results, and gave an almost sure local well-posedness in H^s with $\frac{1}{6} \leq s < \frac{1}{2}$, which covers the lower endpoint. This result is optimal in the strong sense that the Duhamel term belongs to $H^{\frac{1}{2}}$.

Furthermore, we also proved the first probabilistic global result for $3D$ defocusing cubic NLS without imposing any a priori condition or size restriction. More precisely, we proved an almost sure global well-posedness and scattering with radial initial data in H^s with $\frac{3}{7} \leq s < \frac{1}{2}$. Our argument is based on a bootstrap argument under a probabilistic high-low frequency decomposition, combining variants of bilinear Strichartz estimate, a perturbation version of interaction Morawetz estimate, and various kinds of global space-time estimates based on the atom space. The key ingredient is that we are able to cut down the order of derivative more than $\frac{1}{2}$.

Some results on one-dimension quasilinear wave equations

Dongbing Zha (查冬兵)

Donghua University, China

Abstract. In this talk, we will introduce some stability results for one-dimension systems of quasilinear wave equations with null conditions. We will first show the global existence in the small data setting, then prove the global stability of traveling wave solutions, for the Cauchy problem, finally give some global existence results for the initial-boundary value problem in the semilinear case.

Low regularity well-posedness and breakdown criterion of the water wave equation

Zhifei Zhang (章志飞)

Peking University, China

Abstract. It is a long-standing problem whether the water wave equation could develop a finite time singularity. As a first step toward this problem, Craig and Wayne proposed to study the low regularity well-posedness and breakdown criterion of the water wave equation. In this talk, I will introduce some progress about the Craig-Wayne's problem.