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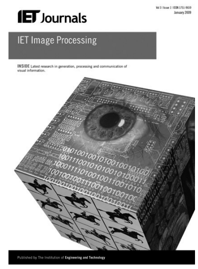

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Patch-based locality-enhanced collaborative representation for face recognition

Ru-Xi Ding[1,2], He Huang[1,2], Jin Shang[3]

[1]Center for Applied Mathematics of Tianjin University, Tianjin 300072, People's Republic of China
[2]Department of Mathematics, School of Science, Tianjin University, Tianjin 300072, People's Republic of China
[3]Center for Combinatorics, LPMC-TJKLC, Nankai University, Tianjin 300071, People's Republic of China
E-mail: huanghe901112@sina.cn

Abstract: In the field of face recognition, the small sample size (SSS) problem and non-ideal situations of facial images are recognised as two of the most challenging issues. Recently, Zhu *et al*. proposed a patch-based collaborative representation (PCRC) method which showed good performance for the SSS and the single sample per person problems; and Peng *et al*. proposed a locality-constrained collaborative representation (LCCR) method which achieved high robustness for face recognition in non-ideal situations. Inspired by the methods proposed in PCRC and LCCR, this study proposes a patch-based locality-enhanced collaborative representation (PLECR) method to combine and enhance the advantages of both PCRC and LCCR. The PLECR and several related methods are implemented on AR, face recognition technology and extended Yale B databases; and the extensive numerical results show that PLECR is more efficient among these methods for the SSS problem in non-ideal situations, especially for the SSS problem with occlusions.

1 Introduction

It was proposed in [1] that natural images can be coded by structural primitives sparsely. With the development of l_0-norm and l_1-norm minimisation techniques [2–4], sparse representation has been widely studied for image restoration [5, 6]. In [7], the sparse representation-based classification (SRC) method for robust face recognition was proposed. The effective performance of SRC has brought sparsity-based pattern classification into widely studied by researchers [8–10].

In spite of the great achievements of sparse representation for classification, the role of sparsity in classification has been widely discussed recently [11–13]. Based on the fact that all the facial images had similar information and structures even though they belonged to different classes, Zhang *et al*. [12] proposed a collaborative representation-based classification (CRC) method for face recognition. The optimal problem in CRC has a closed-form solution and the projection matrix can be computed off-line. The numerical results in [12] showed that CRC can achieve the similar recognition rates as SRC, but reduce the computational cost significantly.

As a widely studied model, manifold learning is usually used to reduce the dimension via reserving some expected local geometric structures [14, 15]. In addition, local consistency [16, 17] means that there are same properties which exist in the nearby data points. By integrating the local consistency and CRC, Peng *et al*. [18] proposed a locality-constrained collaborative representation (LCCR) method for face recognition, which is more robust and precise for face recognition in non-ideal circumstances, such as the facial images varying with expression, illumination, corruptions and occlusions.

Despite the significant achievements of face recognition in computer vision and pattern recognition fields [19], it is still a big challenge to improve the performance for the small sample size (SSS) problem and the non-ideal situations. Patch skill not only performs well for face recognition [20], but also is one of the efficient methods for solving these issues. The main reasons lie in patching the images into small blocks can increase the number of training samples of each subject relatively; and a certain number of blocks of each sample may not include the non-ideal information after being patched. In [21], the patch-based CRC (PCRC) method was proposed to improve the performance of CRC for the SSS problem.

Based on these works, a patch-based locality-enhanced collaborative representation (PLECR) method for face recognition is proposed in this paper. The PLECR patches the training samples and the test sample into overlapped blocks in the same way. For each test block, PLECR finds the nearest neighbourhood from the training set blocks. The optimal combination of the original test block and its nearest neighbour block is considered as a new test sample in the classification. For each test block, the optimisation problem under consideration has a closed-form solution. Therefore the projection matrix can be computed off-line. We compare PLECR with SRC, CRC, LCCR, PCRC and the Block Fisher linear discriminant analysis (FLDA) [22] (which is very effective for solving SSS problem) on AR database (ARD), face recognition technology (FERET) database and extended Yale B database. The extensive experiments show that PLECR is more effective than other five methods for the SSS problem in non-ideal situations.

The rest of this paper is organised as follows. Section 2 describes the related methods. Section 3 presents PLECR method. Section 4 gives experiments of six methods on the standard face databases. Conclusions are contained in Section 5.

2 Brief review of related works

We consider a set of facial training samples which includes C subjects with the kth subject having n_k samples. For any $i \in \{1, 2, \ldots, n_k\}$ and $k \in \{1, 2, \ldots, C\}$, let the grey matrix of the ith training sample in the kth subject be stacked into a vector, denoted by $\boldsymbol{d}_{ik} \in \mathbb{R}^m$. We use $\boldsymbol{D}_k \in \mathbb{R}^{m\times n_k}$ to denote the matrix of training samples in the kth subject, with the ith column of $\boldsymbol{D}_k$ being $\boldsymbol{d}_{ik}$; and use $\boldsymbol{D} = \begin{bmatrix} \boldsymbol{D}_1 & \boldsymbol{D}_2 & \cdots & \boldsymbol{D}_C \end{bmatrix}$ to denote the matrix of all training samples. In addition, the grey matrix of the test sample is stacked into a vector, denoted by $\boldsymbol{x} \in \mathbb{R}^m$.

In SRC, each test sample $\boldsymbol{x}$ is coded by $\boldsymbol{D}$ via a spare coefficient α

$$\min_{\alpha} \| \alpha \|_0 \quad \text{s.t.} \quad \boldsymbol{x} = \boldsymbol{D}\alpha \tag{1}$$

where $\|\cdot\|_0$ is l_0-norm. Since (1) is an NP-hard problem, one considers its relaxation model and solves the following optimisation problem

$$\min_{\alpha} \|\boldsymbol{x} - \boldsymbol{D}\alpha\|_2^2 + \lambda \|\alpha\|_1 \tag{2}$$

where $\lambda \in (0, 1)$ is a parameter. Let $\alpha^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_C^*)^{\mathrm{T}}$ be the optimal solution of (2) with $\alpha_k^* \in \mathbb{R}^{n_k}$ being the corresponding coefficient vector in α^* of the kth subject. Then the classification of SRC is designed by identity($\boldsymbol{x}$) = $\mathrm{argmin}_k r_k$ with $r_k = \|\boldsymbol{x} - \boldsymbol{D}_k\alpha_k^*\|_2$ and $k \in \{1, 2, \ldots, C\}$, where identity($\boldsymbol{x}$) denotes the label for $\boldsymbol{x}$.

CRC codes the test sample $\boldsymbol{x}$ by $\boldsymbol{D}$ via l_2-norm regularisation, i.e.

$$\min_{\alpha} \|\boldsymbol{x} - \boldsymbol{D}\alpha\|_2^2 + \lambda \|\alpha\|_2^2 \tag{3}$$

where λ is a regularisation parameter. The optimal solution of (3) is given by $\alpha^* = \boldsymbol{P}\boldsymbol{x}$ where the projection matrix $\boldsymbol{P} := (\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D} + \lambda\cdot\boldsymbol{I})^{-1}\boldsymbol{D}^{\mathrm{T}}$. Since $\boldsymbol{P}$ is independent of the test sample $\boldsymbol{x}$, it can be computed off-line. This is the reason why CRC is dramatically faster than SRC. The classification of CRC is the same as SRC by identity($\boldsymbol{x}$) = $\mathrm{argmin}_k r_k$; but each residual r_k is computed by $r_k = \|\boldsymbol{x} - \boldsymbol{D}_k\alpha_k^*\|_2 / \|\alpha_k^*\|_2$ where $k \in \{1, 2, \ldots, C\}$ and $\alpha_k^* \in \mathbb{R}^{n_k}$ is the coefficient vector of the kth subject in α^*.

LCCR achieves high robustness by using the neighbourhoods $\boldsymbol{Y}(\boldsymbol{x})$ to replace the test sample $\boldsymbol{x}$ to some extent. $\boldsymbol{Y}(\boldsymbol{x}) \in \mathbb{R}^{m\times K}$ includes K neighbourhoods of test sample $\boldsymbol{x}$, which is selected from the training set $\boldsymbol{D}$. The model of LCCR is given by

$$\min_{\alpha} (1-\gamma) \| \boldsymbol{x} - \boldsymbol{D}\alpha \|_2^2 + \gamma \frac{1}{K} \sum_{\boldsymbol{y}_{h(x)} \in \boldsymbol{Y}(\boldsymbol{x})} \|\boldsymbol{y}_h(\boldsymbol{x}) - \boldsymbol{D}\alpha\|_2^2 + \lambda \|\alpha\|_2^2 \tag{4}$$

where $\boldsymbol{y}_h(\boldsymbol{x}) \in \mathbb{R}^m$ is the hth neighbourhood for any $h \in \{1, 2, \ldots, K\}$; λ is a regularisation parameter; and $\gamma \in [0, 1]$ is a balance parameter. The solution of (4) is given by $\alpha^* = \boldsymbol{P}[(1-\gamma)\boldsymbol{x} + \gamma(1/K)\sum_{\boldsymbol{y}_h(\boldsymbol{x})\in\boldsymbol{Y}(\boldsymbol{x})} \boldsymbol{y}_h(\boldsymbol{x})]$, where $\boldsymbol{P} := (\boldsymbol{D}^{\mathrm{T}}\boldsymbol{D} + \lambda\cdot\boldsymbol{I})^{-1}\boldsymbol{D}^{\mathrm{T}}$. The regularised residual r_k is computed by the same way as the one in CRC, then the recognition result of test sample $\boldsymbol{x}$ is obtained by using identity($\boldsymbol{x}$) = $\mathrm{argmin}_k r_k$.

In [21], patch skill was introduced into CRC and the corresponding method was denoted as PCRC. The patch skill can be summarised as follows: given a test sample $\boldsymbol{x}$, first divide it into J block patches with $\boldsymbol{x}_j$ being the jth block of $\boldsymbol{x}$, where $j \in \{1, 2, \ldots, J\}$. For any $i \in \{1, 2, \ldots, n_k\}$ and $k \in \{1, 2, \ldots, C\}$, each training sample $\boldsymbol{d}_{ik}$ is divided into the same block patches with $\boldsymbol{d}_{ikj}$ being the jth block of $\boldsymbol{d}_{ik}$ for all $j \in \{1, 2, \ldots, J\}$. For any $j \in \{1, 2, \ldots, J\}$ and $k \in \{1, 2, \ldots, C\}$, we denote $\boldsymbol{D}_{kj} := \begin{bmatrix} \boldsymbol{d}_{1kj} & \boldsymbol{d}_{2kj} & \cdots & \boldsymbol{d}_{n_k kj} \end{bmatrix}$, and then $\boldsymbol{D}_j := \begin{bmatrix} \boldsymbol{D}_{1j} & \boldsymbol{D}_{2j} & \cdots & \boldsymbol{D}_{Cj} \end{bmatrix}$. Then, for any $j \in \{1, 2, \ldots, J\}$, PCRC is to solve

$$\min_{\alpha_j} \|\boldsymbol{x}_j - \boldsymbol{D}_j\alpha_j\|_2^2 + \lambda \|\alpha_j\|_2^2$$

and its optimal solution is $\alpha_j^* = (\boldsymbol{D}_j^{\mathrm{T}}\boldsymbol{D}_j + \lambda\cdot\boldsymbol{I}_j)^{-1}\boldsymbol{D}_j^{\mathrm{T}}\boldsymbol{x}_j$. Then compute the residual for each patch by $r_{kj} = \|\boldsymbol{x}_j - \boldsymbol{D}_{kj}\alpha_{kj}^*\|_2 / \|\alpha_{kj}^*\|_2$, where $\alpha_{kj}^* \in \mathbb{R}^{n_k}$ is the coefficient vector in α^* of the jth block in the kth subject. The class of the patch $\boldsymbol{x}_j$ is given by identity($\boldsymbol{x}_j$) = $\mathrm{argmin}_k r_{kj}$. Finally, the class of the test sample is decided by the majority vote of all patches.

3 Patch-based locality-enhanced collaborative representation

In this section, we introduce PLECR method for face recognition, which is described as follows.

We first patch the test sample $\boldsymbol{x}$ into J blocks (Our patch method is illustrated in the following Fig. 1.). Let $\boldsymbol{x}_j$ be the jth block of $\boldsymbol{x}$, where $j \in \{1, 2, \ldots, J\}$. For training set $\boldsymbol{D}$, we patch it in the same way as the test sample. For any $i \in$

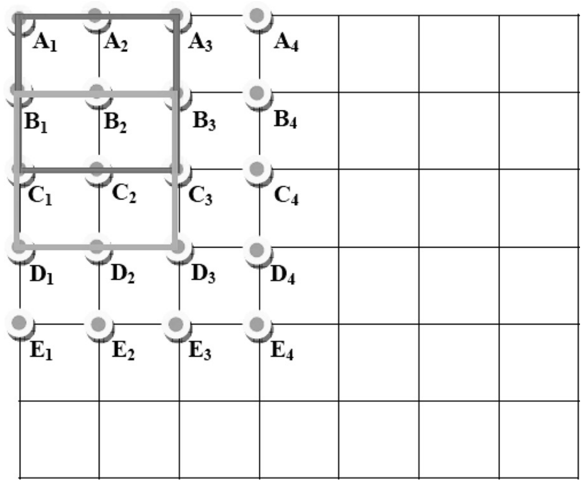

Fig. 1 *Patch method we used is a overlapped patch one*

The first block of the image is Square $A_1A_3C_3C_1$ (denoted as block B_2). The two blocks around block B_2 in the vertical and horizontal directions are Square $B_1B_3D_3D_1$ (denoted as block C_2) and Square $A_2A_4C_4C_2$ (denoted as block B_3), respectively. Rectangle $A_2A_3C_3C_2$ is the overlapped area of block B_2 and block B_3. Rectangle $B_1B_3C_3C_1$ is the overlapped area of block B_2 and block C_2. The length between Point A_2 and Point A_3 is the fixed overlapped size, which is equal to the length between Point B_1 and Point C_1

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$\{1, 2, \ldots, n_k\}$ and $k \in \{1, 2, \ldots, C\}$, the block $\boldsymbol{d}_{ikj}$ is the jth block of $\boldsymbol{d}_{ik}$, where $j \in \{1, 2, \ldots, J\}$. For any $k \in \{1, 2, \ldots, C\}$ and $j \in \{1, 2, \ldots, J\}$, we denote $\boldsymbol{D}_{kj} := \begin{bmatrix} \boldsymbol{d}_{1kj} & \boldsymbol{d}_{2kj} & \cdots & \boldsymbol{d}_{n_k kj} \end{bmatrix}$, and then $\boldsymbol{D}_j := \begin{bmatrix} \boldsymbol{D}_{1j} & \boldsymbol{D}_{2j} & \cdots & \boldsymbol{D}_{Cj} \end{bmatrix}$. Then for each test block $\boldsymbol{x}_j$, we use the nearest neighbour algorithm to find the nearest neighbours $\boldsymbol{y}_j$ from the same block of all training set $\boldsymbol{D}_j$ with l^1-distance. The optimal problem is as follows

$$\min_{\alpha_j} \| (1-\gamma)\boldsymbol{x}_j + \gamma \boldsymbol{y}_j - \boldsymbol{D}_j \alpha_j \|_2^2 + \lambda \| \alpha_j \|_2^2 \qquad (5)$$

It is easy to obtain that the optimal solution of (5) is

$$\alpha_j^* = \boldsymbol{P}_j \Big[(1-\gamma)\boldsymbol{x}_j + \gamma \boldsymbol{y}_j \Big] \qquad (6)$$

where $\boldsymbol{P}_j := (\boldsymbol{D}_j^{\mathrm{T}} \boldsymbol{D}_j + \lambda \cdot \boldsymbol{I}_j)^{-1} \boldsymbol{D}_j^{\mathrm{T}}$. Let α_{kj}^* be the corresponding coefficient vector in α^* of the jth block in the kth subject. Compute the regularised residual of the jth block between the test sample and the kth subject by

$$r_{kj} = \| \boldsymbol{x}_j - \boldsymbol{D}_{kj} \alpha_{kj}^* \|_2 \,/\, \| \alpha_{kj}^* \|_2 \qquad (7)$$

and compute the identity of the jth block by

$$z_j = \text{identity}(\boldsymbol{x}_j) = \operatorname*{argmin}_k \; r_{kj} \qquad (8)$$

Finally, the class of the test sample $\boldsymbol{x}$ is decided by the majority vote of all blocks. Therefore the PLECR method can be summarised as follows.

Algorithm 3.1: the PLECR method

1. Patch each training sample and the test sample $\boldsymbol{x}$ into overlapped blocks in the same way.
2. Normalise the columns of all the patched test blocks and training blocks to have unit l_2-norm.
3. For any $j \in \{1, 2, \ldots, J\}$, compute α_j^* by (6).
4. For any $j \in \{1, 2, \ldots, J\}$ and $k \in \{1, 2, \ldots, C\}$, compute r_{kj} by (7).
5. For any $j \in \{1, 2, \ldots, J\}$, find the identity of the jth block z_j by (8).
6. Output the final identity of the test sample $\boldsymbol{x}$ via majority voting in $\{z_1, z_2, \ldots, z_C\}$.

4 Numerical experiments

In this section, we compare PLECR with SRC [7], CRC [12], LCCR [18], PCRC [21] and BlockFLDA [22] on three standard face databases: AR database [23], FERET database [24] and extended Yale B database [25]. All experiments are finished in MATLAB 64 bit on a Lenovo Think Center M9201z workstation with an Intel Core i5-3550S 2.9 GHz central processing unit and 8 GB of random access memory.

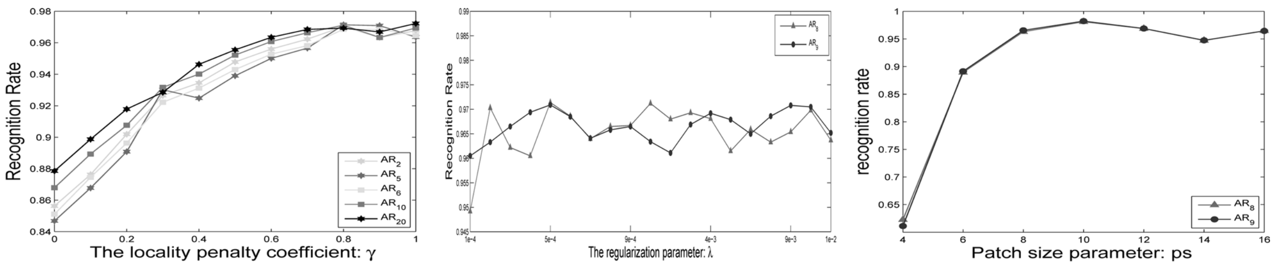
For experiments, the l_1-regularised minimisation in SRC is solved by $l_{1}\_l_s$ method; all the methods have been implemented without dimensionality reduction except for BlockFLDA; in BlockFLDA, the dimension after reduction is the size of training set class C (mentioned in Section 2); the experiments of parameters setting are carried on the three databases for PLECR and other compared methods, except for SRC, because of the expensive computing cost; the parameter λ has been set 0.001 in SRC throughout all experiments.

4.1 Experiments on AR face database

In this section, we aim at to inspect the effectiveness of PLECR for frontal SSS face recognition with complicated varying conditions. Since AR face database contains 3276 colour images with different illuminations (all side lights on, left light on and right light on), varying facial expressions (neutral, smile, anger and scream) and disguise (sunglasses and scarf) from 70 males and 56 females, we elect AR database to be implemented in this section. Each subject on AR database includes 26 images. A subset, containing 50 male subjects and 50 female subjects with 26 images per subject, is selected for the experiments, which signed as ARs. All images are cropped from original 165 × 120 to 50 × 40 grey scale.

As an example for experiments of parameters setting, the PLECR is implemented on ARs, we choose randomly 13 samples of each subject for training and the rest 13 samples of each subject for testing. The experiments are randomly done ten times and the average recognition rates are recorded.

- The patch size $ps = 8$, presented by Zhu *et al.* [21], was the best when the effect of patch size was discussed on extended Yale B. Based on these, by setting $ps = 8$, we show the recognition rate against the value of the locality penalty coefficient γ in Fig. 2*a*. The different lines show the variation of the recognition rate against γ by fixing different values of the parameter λ: $\text{AR}_2(\lambda = 0.0002)$, $\text{AR}_5(\lambda = 0.0005)$, $\text{AR}_6(\lambda = 0.0006)$, $\text{AR}_{10}(\lambda = 0.001)$ and $\text{AR}_{20}(\lambda = 0.002)$. From Fig. 2*a*, it seems that $\gamma = 0.8$ is the best one for ARs.
- In Fig. 2*b*, by setting $ps = 8$, we present the variation of the recognition rate against λ with $\gamma = 0.8$, 0.9. It seems that $\lambda = 0.009$ is the best one for ARs with $\gamma = 0.8$.
- By setting $\lambda = 0.009$, we propose the variation of the recognition rate against ps in Fig. 2*c* with $\gamma = 0.8(\text{AR}_8)$ and



a *b* *c*

Fig. 2 *Recognition results of varying parameters of PLECR on ARs*

a The recognition rate of PLECR with varying locality penalty coefficient γ
b The recognition rate of PLECR with varying regularisation parameter λ
c The recognition rate of PLECR with varying patch size parameters *PS*

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Table 1 Average recognition rates (%) on ARs database

Methods	2/5	3/5	4/5	5/5	13/13
SRC	75.79	85.16	87.08	90.93	92.41
CRC	77.80	77.36	86.83	90.39	90.70
LCCR	74.01	84.63	85.97	91.84	95.60
PCRC	66.38	71.25	79.91	80.68	90.47
BlockFLDA	61.04	69.33	78.14	81.67	90.38
PLECR	**80.07**	**85.17**	**89.64**	**93.16**	**98.20**

$\gamma = 0.9(AR_9)$. It seems that $ps = 10$ with overlap size $po = 5$ is the best one for ARs.

According to Fig. 2, the parameters are set as $\gamma = 0.8$, $\lambda = 0.009$, $ps = 10$ and $po = 5$ for PLECR in our experiments on AR database. The same experiments have been done for other compared methods on AR database. Depending on the results of these experiments, the parameters are set as: $\lambda = 0.001$ in CRC; $\lambda = 0.001$, $\gamma = 0.4$ and $K = 4$ in LCCR; $\lambda = 0.001$ and $ps = 16$ with $po = 8$ in PCRC; and $\lambda = 0.01$ and $ps = 6$ in BlockFLDA.

Now we implement PLECR and other five methods on ARs database for frontal SSS face recognition with complicated varying conditions. We choose randomly 2, 3, 4, 5 samples of each subject, for training; and choose randomly 5 samples from the rest of each subject for testing. In addition, we also choose randomly 13 samples of each subject for training and the rest 13 samples of each subject for testing. The experiments are randomly done 20 times for each tested problem, and the average recognition rates are showed in Table 1. In Table 1, *a*/*b* means *a* samples for training and *b* samples for testing (Note: the same symbol is used in the rest of tables.).

From Table 1, it is easy to see that the recognition rates of PLECR are higher than other five related methods on ARs database. It also can be concluded that PLECR is effective for both SSS problem and normal size frontal-face recognition with complicated varying conditions, such as varying illumination, expression and occlusion.

4.2 Experiments on FERET face database

In this section, the experiment is to verify the performance of PLECR for SSS problem with change of pose and expression. Since there are 200 subjects of FERET database [24] with some varying pose and expression conditions, we use FERET database in this section. For the 200 subjects, there are seven grey images per subject with cropped size 80 × 80. The samples of the first three subjects are showed in Fig. 3.

The experiments of parameters setting, which are analogous to the one in Section 4.1, are done on the 200 subjects. According to the average values of 20 times experiments results, the parameters are set as: $\gamma = 0.5$, $\lambda = 0.00005$, $ps = 16$ and $po = 8$ in PLECR; $\lambda = 0.001$ in CRC; $\lambda = 0.005$, $\gamma = 0.4$ and $K = 2$ in LCCR; $\lambda = 0.0005$ and $ps = 12$ with $po = 6$ in PCRC; and $\lambda = 0.05$ and $ps = 3$ in BlockFLDA.

Now, we carry out the experiment of the concerned six methods for SSS problem with change of pose and expression on FERET database. The first 2, 3, 4, 5 samples have been chosen for training, and the rest samples for testing. Table 2 shows the recognition results on FERET database.

As the numerical results showed in Table 2, PLECR is effective for SSS face recognition with varying pose and expression conditions. In Table 2, the recognition rates of PCRC are lower than those of CRC; the recognition rates of LCCR are higher than CRC except for the number of training samples is 5, but the recognition rates of PLECR are higher than LCCR. The reasons are explained as follows. PCRC is the incorporation of CRC and patch skill directly. Since the patch skill is not very effective for the pose changing and the seven samples of each subject on FERET database contain different pose conditions, the performances of CRC and PCRC on FERET database are reasonable. In PLECR, the practical testing part is the optimal combination of the testing block and its nearest neighbour of testing blocks, which improves the robustness of PLECR for the pose changing. Meanwhile, the relation of PLECR and LCCR is not the same way as PCRC and CRC. PLECR just needs the nearest neighbour of testing sample, but LCCR needs to find more than one neighbour. As the difference of each sample in the same subject is dramatic, the less neighbours used, the better performance

Table 2 Recognition rates (%) on FERET database

Methods	2/5	3/4	4/3	5/2
SRC	62.90	58.25	68.17	79.50
CRC	46.7	39.00	45.83	60.75
LCCR	60.50	53.00	62.83	53.25
PCRC	37.80	35.25	38.00	48.75
BlockFLDA	56.30	50.88	59.33	65.75
PLECR	**63.30**	**59.00**	**69.33**	**80.00**

Fig. 3 *Samples of the first three subjects in FERET database*

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Fig. 4 *All samples of the first subject in extended Yale B database*

Table 3 Recognition rates (%) on extended Yale B database

Methods	2/20	3/20	4/20	5/20	6/20	7/20
SRC	26.91	33.53	36.51	44.26	43.01	45.28
CRC	19.63	31.78	34.86	41.78	44.11	44.92
LCCR	20.03	32.87	33.47	42.79	43.76	45.28
PCRC	25.88	**38.74**	41.21	49.66	50.80	51.74
BlockFLDA	11.36	23.39	33.93	39.01	40.55	44.37
PLECR	**34.01**	37.46	**47.88**	**51.50**	**52.33**	**54.11**

would be obtained. Therefore the recognition rates showed in Table 2 are reasonable.

4.3 Experiments on extended Yale B face database

In this section, the experiment focuses on whether PLECR is efficient for frontal SSS problem with obvious variation on illumination. Extended Yale B face database [25] has been considered, since it includes 2414 frontal-face images with 64 illumination conditions of size 192×168 from 38 subjects. In our experiment, we choose 64 normalised images each subject with cropped size 80×80, as showed in Fig. 4.

For experiments of parameters setting, we choose randomly 32 samples per subject for training and the rest 32 samples for testing with 20 times cycling. According to the average numerical results, the parameters are set as: $\gamma = 0.05$, $\lambda = 0.00005$, $ps = 12$ and $po = 6$ in PLECR; $\lambda = 0.001$ in CRC; $\lambda = 0.05$, $\gamma = 0.2$ and $K = 4$ in LCCR; $\lambda = 0.00005$ and $ps = 16$ with $po = 8$ in PCRC; and $\lambda = 0.001$ and $ps = 14$ in BlockFLDA in the following experiments on extended Yale B database.

Now, we implement PLECR and other five methods for the above database. For every subject, we choose randomly 2, 3, 4, 5, 6, 7 samples for training and 20 samples from the rest for testing with ten times for obtaining the average recognition rates. The recognition rates are showed in Table 3.

As showed in Table 3, we can conclude that PLECR is effective for frontal SSS problem with broad change in illumination.

4.4 Experiments for images with occlusions

In this section, the six methods are implemented on two subsets of ARs database with different occlusion situations, where all parameters are set as the same as those given in Section 4.1. The experiments are divided into the following two parts. In the first part, the main purpose is to verify the performance of PLECR on a database which includes occlusion images per subject. In the other part, more extended experiments have been done to check the performance of PLECR for SSS problem with artificially added different occlusion situations.

4.4.1 Part I: experiments for images with occlusions from ARs: In this part, ARs database is further divided into two subsets: ARD which includes 12 images per subject with some occlusions such as wearing sunglasses or scarf; and ARE which includes the rest 14 images per subject with varying expression and illumination, as showed in Fig. 5.

Now, we implement the considered six methods on ARD database. In our experiments, for every subject, we choose randomly 2, 3, 4, 5 samples for training, and choose randomly 5 samples from the rest samples for testing; we also choose randomly 6 samples for training and the rest 6 samples for testing. The average recognition rates for 20 times' experiments of each case are showed in Table 4.

From Table 4, it is easy to see that PLECR outperforms other methods for the experiments on ARD database, which indicates effectiveness of PLECR for face recognition with occluded conditions.

Fig. 5 *Samples in ARD and ARE databases*

a Shows all the samples of the first subject in ARD
b Shows all the samples of the same subject in ARE

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Table 4 Average recognition rates (%) on ARD database

Methods	2/5	3/5	4/5	5/5	6/6
SRC	64.88	79.77	79.12	**90.72**	91.97
CRC	69.05	79.63	85.74	81.27	90.11
LCCR	67.16	75.04	83.33	88.24	89.19
PCRC	58.52	67.74	76.18	81.03	87.21
BlockFLDA	57.08	66.31	75.48	78.55	86.50
PLECR	**70.88**	**81.64**	**86.17**	**90.72**	**95.11**

Table 5 Average recognition rates (%) on ARE database

Methods	2/12	3/11	4/10	5/9	6/8	7/7
SRC	**79.00**	**79.73**	78.00	77.44	76.75	75.29
CRC	76.83	75.18	72.70	72.11	70.50	65.71
LCCR	75.17	74.36	71.00	77.00	77.25	75.00
PCRC	63.75	66.91	68.90	69.67	67.63	61.86
BlockFLDA	63.08	65.73	65.20	68.78	69.88	67.57
PLECR	77.58	78.18	**79.40**	**81.33**	**81.25**	**78.86**

a b

Fig. 6 *Figure shows the first sample in ARE with different block occlusions varying from 20 to 60%*

a Shows the samples with block occlusion from bottom to top
b Shows the samples with block occlusion from top to bottom

4.4.2 Part II: experiments for images with different occlusions on AR expression database: The samples of ARE database have varying expression and illumination. To check the performance of PLECR with occlusion conditions sufficiently, we do more experiments on the AR expression (ARE) database. The experiments are further divided into three sub-parts. The first is a contrast experiment without any added occlusion on ARE database; and other two sub-parts are experiments on ARE database by adding different occlusions, including black occlusion from bottom to top and black occlusion from top to bottom. In the following, we show these experiments one by one.

4.4.3 Part II.a: a contrast experiment on ARE: As a contrast experiment on ARE, we choose the first 2, 3, 4, 5, 6, 7 samples of each subject, for training; and the rest samples of each subject for testing. The numerical results are showed in Table 5.

As showed in Table 5, PLECR performs better than other five methods when the number of training samples is larger than 3.

In the following four sub-parts, we test the concerned six methods on ARE database with different occlusions. In every experiment, we choose the first 2, 3, 4, 5, 6, 7 samples of each subject, for training; and the rest samples of each subject with added four different occlusions for testing.

4.4.4 Part II.b: experiments for images with black occlusion form bottom to top: In this sub-part's experiment, the black occlusion is added to each test sample from bottom to top with the occluded rate varying from 20 to 60%, as showed in Fig. 6*a*. The recognition rates are showed in Table 6.

From Table 6, it is easy to see that PLECR is more effective and stable than other five methods with different occlusion ratios in this situation.

4.4.5 Part II.c: experiments for images with black occlusion from top to bottom: We carry out the experiment on ARE database by adding the black occlusions to each test sample from top to bottom with the occluded ratio varying from 20 to 60%, as showed in Fig. 6*b*. The numerical results are showed in Table 7.

Compared Table 7 with Table 6, the performances of the six methods have some decrease to different extent in experiments with adding black occlusion from top to bottom. The reasons are that the distinguishing information (eyebrows, eyes, nose and mouse) concentrates on upper part of face; and the occlusion has been added from top to bottom. Moreover, the recognition rates of PLECR drops the lowest. From Table 7, we conclude that PLECR is more effective than other five methods with the varying black occlusion added from top to bottom.

Compared Tables 6 and 7 with Table 5, it is obvious that PLECR performs more effective and stable than other five related methods with all the added occlusion conditions on ARE database. As mentioned above, the images in ARE database include some non-ideal information. Thus, the

Table 6 Recognition rates (%) with black occlusions from bottom to top

Perc	Methods	2/12	3/11	4/10	5/9	6/8	7/7
0.2	SRC	58.83	57.55	56.70	58.33	53.13	48.57
	CRC	50.92	47.09	52.30	53.00	50.50	39.71
	LCCR	55.25	52.45	52.70	54.67	54.25	47.14
	PCRC	65.83	70.36	69.80	69.11	66.25	59.71
	BlockFLDA	63.33	64.36	64.20	65.44	65.38	62.14
	PLECR	**76.00**	**74.00**	**75.40**	**79.56**	**79.75**	**77.29**
0.4	SRC	37.42	39.00	34.70	29.89	38.50	30.29
	CRC	40.33	42.82	36.00	38.00	37.00	23.14
	LCCR	33.83	34.64	29.40	31.22	31.13	22.57
	PCRC	59.83	62.82	64.50	62.67	59.13	50.14
	BlockFLDA	59.42	58.36	54.00	60.56	57.88	55.00
	PLECR	**68.50**	**71.55**	**72.20**	**75.11**	**75.00**	**71.86**
0.6	SRC	29.75	33.18	26.80	24.89	27.13	22.57
	CRC	26.33	33.00	29.80	25.56	24.50	18.00
	LCCR	21.25	24.36	22.70	20.67	18.88	12.86
	PCRC	35.92	50.09	49.00	44.00	38.38	31.57
	BlockFLDA	44.50	44.45	40.10	45.67	42.88	36.29
	PLECR	**57.00**	**62.09**	**62.50**	**66.89**	**63.50**	**44.52**

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Table 7 Recognition rates (%) with black occlusions from top to bottom

Perc	Methods	2/12	3/11	4/10	5/9	6/8	7/7
0.2	SRC	31.17	25.55	22.40	25.11	30.13	25.71
	CRC	55.42	54.00	54.10	54.56	56.88	52.00
	LCCR	51.00	47.00	43.70	49.00	54.38	48.14
	PCRC	51.33	52.00	55.20	59.33	54.88	47.71
	BlockFLDA	49.33	53.55	54.40	60.33	62.38	58.57
	PLECR	**67.92**	**71.18**	**72.00**	**75.11**	**74.25**	**71.43**
0.4	SRC	11.67	8.82	8.80	8.67	9.38	9.00
	CRC	26.17	34.55	36.80	42.00	46.88	39.86
	LCCR	28.83	26.36	25.00	29.33	30.25	23.43
	PCRC	41.75	42.82	45.40	47.78	46.88	40.14
	BlockFLDA	14.75	37.09	37.60	45.56	49.38	43.71
	PLECR	**58.75**	**62.64**	**63.50**	**67.11**	**64.13**	**60.71**
0.6	SRC	4.83	4.36	3.80	3.89	3.63	3.57
	CRC	20.17	19.00	15.50	19.56	23.00	15.14
	LCCR	17.75	12.18	11.80	11.00	11.38	9.29
	PCRC	12.50	16.18	20.00	26.89	10.13	12.00
	BlockFLDA	5.33	17.64	18.10	23.78	30.88	24.00
	PLECR	**45.08**	**49.73**	**45.70**	**50.22**	**47.25**	**31.71**

conclusion of this section is that PLECR is more effective than other five methods for solving the frontal SSS problem in non-ideal circumstances, especially for the SSS problem with occluded conditions.

5 Conclusions

This paper focuses on improving performance of the face recognition method for SSS problem in non-ideal circumstances. A PLECR was proposed for face recognition, and the extensive numerical experiments on AR, FERET and extended Yale B databases were done by comparing PLECR with SRC, CRC, LCCR, PCRC and BlockFLDA. The experiments results showed that PLECR is more efficient and robust than other five methods for SSS problems in non-ideal situations, especially for SSS problems with different occlusion conditions.

Since the patch size has big influence in the proposed method, it is possible that some multi-scale patch method can be added to this method to improve the robustness of patch size.

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