

Accurate Network Alignment via Consistency in Node Evolution

Yinghui Wang*, Qiyao Peng*, Pengfei Jiao, *Member, IEEE*, Huaming Wu, *Senior Member, IEEE*, and Wenjun Wang

Abstract—Network alignment, which integrates multiple network resources by identifying anchor nodes that exist in different networks, is beneficial for conducting comprehensive network analysis. Although there have been many studies on network alignment, most of them are limited to static scenarios and only can achieve acceptable top- α ($\alpha > 10$) results. In the absence of considering dynamic changes in networks, accurate network alignment (i.e., top-1 result) faces two problems: 1) Missing information: focusing solely on aligning networks at a specific time leads to low top-1 performance due to the lack of information from other time periods; 2) Confusing information: ignoring temporal information and focusing on aligning networks across the entire time span leads to low top-1 performance due to inability to distinguish the neighborhood nodes of anchor nodes. In this paper, we propose a dynamic network alignment method, which aims to achieve better top-1 alignment results with consider changing network structures over time. Towards this end, we learn the representations of nodes in the changing network structure with time, and preserve the consistency of anchor node pairs during the time-evolution process. Firstly, we employ a Structure-Time-aware module to capture network dynamics while preserving network structure and learning node representations that incorporate temporal information. Secondly, we ensure the global and local consistency of anchor node pairs over time by utilizing linear and similarity functions, respectively. Finally, we determine whether two nodes are anchor node pairs by maintaining consistency between global, local, and node representations. Experimental results obtained from real-world datasets demonstrate that the proposed model achieves performance comparable to several state-of-the-art methods.

Index Terms—Network alignment, network evolution, global consistency, local consistency.

1 INTRODUCTION

NETWORK alignment is a process that compares two networks to identify common nodes. It has gained attention for its broad applications, such as comparing gene networks or protein-protein interaction networks in biology [1], [2], collecting the accounts belonging to the same person in different online platforms in sociology [3], [4], inferring the cross-layer alignment of wired and wireless networks in computer science [5], [6], inferring relationships among entities from different sources and to facilitate transfer learning in knowledge graphs [7], [8], [9], [10]. The common nodes found in different networks are referred to as anchor nodes [11]. Anchor nodes represent entities such as the accounts of the same person across different social networks. The correspondence between pairs of anchor nodes, which represent the same entity, is known as anchor links.

Network alignment tasks have been thoroughly investigated and many methods have been proposed [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]. When predicting anchor links, a majority of these

methods hold a consistency assumption, i.e., the higher the percentage of shared nodes, the more likely two nodes are corresponding anchor nodes. However, most existing research focuses on the alignment between static networks, ignoring the dynamic evolution of the network structure in real-world scenarios.

The dynamic evolution of network structures refers to the appearance of new nodes and/or edges and the disappearance of existing nodes and/or edges over time. Typically, the dynamic evolution of network structures can be represented by a series of static networks: $G = \{G^1, G^2, \dots, G^t, \dots, G^T\}$, G^t can be interpreted as a snapshot captured at a specific moment in time t . While executing static network alignment methods directly on G or G^t can produce respectable top- α ($\alpha > 10$ generally) prediction accuracy, achieving satisfactory top-1 accuracy is often challenging. This is due to the following factors:

(1) Missing information. Static alignment methods usually have poor top-1 accuracy on snapshot network pairs due to snapshots only retaining part of the network information. As shown in Fig. 1(a), the consistency-based method will misalign v_i^1 and v_j^2 at the timestamps $t = t_2$ and t_3 , because v_i^1 and v_j^2 share more anchor node pairs as neighbors between snapshot networks of G^1 and G^2 at $t = t_2$ and t_3 .

(2) Confusing information. When ignoring the timing information of dynamic networks and merging all snapshot networks as a static network, directly applying static alignment methods may have poor top-1 accuracy due to confusing local structure. As shown in Fig. 1(b), it is confusing for v_i^1 in G^1 to judge whether its corresponding

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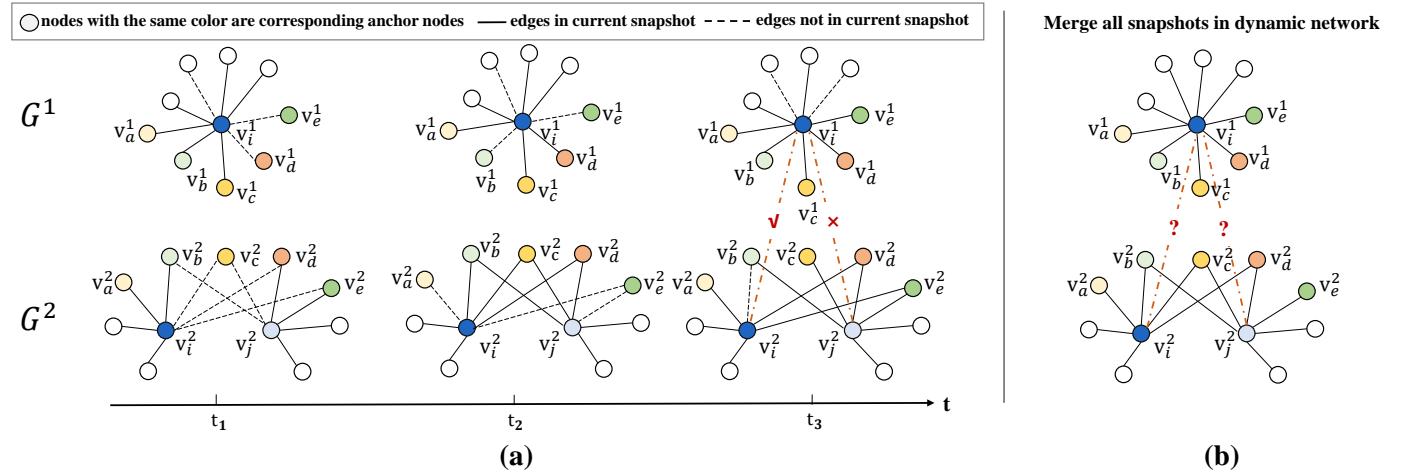


Fig. 1: Illustration of network alignment with/without dynamic evolution of network structures. Nodes with different colors are corresponding anchor nodes that are known in advance. (a) In the dynamic scenario, it can determine the corresponding anchor node of v_i^1 is v_i^2 at t_3 according to their similar neighborhood change pattern. (b) In the static scenario, it is difficult to determine the corresponding anchor node of v_i^1 since v_i^2 and v_j^2 both have a similar neighborhood with v_i^1 .

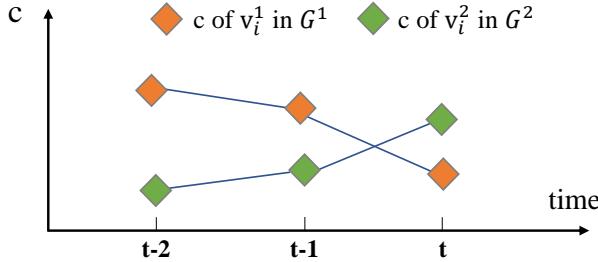


Fig. 2: Illustration of the global consistency of anchor node pairs. c is the number of the first-order neighbors of nodes. With the evolution of G^1 and G^2 , the neighbor number of nodes v_i^1 and v_i^2 in G^1 and G^2 changes differently, but the total neighbor number of v_i^1 and v_i^2 in the two networks obeys a certain pattern.

anchor node is v_i^2 or v_j^2 in G^2 . Since v_i^1 and v_i^2 have the same number of shared nodes as v_i^1 and v_j^2 , and v_i^2 and v_j^2 have similar local structures, it is difficult to make a distinction between v_i^2 and v_j^2 without more auxiliary information.

In order to achieve better top-1 performance, it is crucial to implement a more effective strategy for differentiating neighboring nodes. The dynamic evolution of the network can provide more information. As shown in Fig. 1(a), v_i^1 and v_i^2 share anchor nodes at three timestamps (t_1 , t_2 , and t_3) in the networks G^1 and G^2 . The shared anchor nodes change over time from (v_a^1, v_a^2) , (v_b^1, v_b^2) , (v_c^1, v_c^2) to (v_d^1, v_d^2) , (v_e^1, v_e^2) . Therefore, at $t = t_3$, it can be determined that the corresponding anchor node of v_i^1 is v_i^2 instead of v_j^2 . Therefore, leveraging the evolution information of networks can effectively distinguish nodes within the network and consequently achieve superior network alignment performance.

To improve the top-1 performance of network alignment, we propose a novel alignment model called GLDyNA. This model takes into account not only the dynamic evolution of

the network structure but also the consistent evolution of anchor node pairs in both global and local neighborhoods. For local consistency, its implication is similar to the common assumption in static network alignment methods, i.e., in each pair of snapshot networks, anchor node pairs need to maintain a similar local structure. For global consistency, it reveals the inherent evolution pattern of anchor node pairs over time and imposes constraints at a higher structural level in network alignment, i.e., it determines the number of neighbors an entity should maintain relationships with across different networks. Due to the evolution of network scale over time, there are fluctuations in the number of first-order neighbors, denoted as c , of anchor nodes across different networks, as shown in Fig. 2. Despite the dissimilar variations in the number of first-order neighbors of anchor nodes within networks G^1 and G^2 , there exists a latent regularity in the total number of first-order neighbors of anchor nodes in these networks.

Specifically, to model the dynamics and preserve the network structure within a single network, we introduce a Structure-Time-aware module. This module randomly samples sub-networks of a given node v_i , which include the neighbors of v_i from different snapshots. It then learns representations of v_i by maximizing the probability of co-occurrence of two nodes within these sub-networks. In order to capture the global consistency of anchor node pairs, we define a linear function that relates to the total number of neighbors of anchor node pairs across different networks. This function imposes constraints on how the neighborhood scale of the anchor node pair evolves with the network. To capture the local consistency of anchor node pairs, we define a similarity equation based on structural equivalence. Finally, we combine the global and local consistency, along with the similarity of node representations learned from different networks, to predict anchor node pairs.

The main contributions of this paper could be summarized as follows:

- 110 We analyze why static network alignment methods
111 cannot be directly applied in dynamic scenarios.
112 Additionally, we explain the factors contributing to
113 the poor performance of existing methods in terms
114 of top-1 precision.
- 115 We propose a novel dynamic network alignment
116 method GLDyNA, which takes into account both the
117 influence of network evolution and the global-local
118 consistency of anchor node pairs.
- 119 We evaluate the proposed GLDyNA on different real-
120 world datasets. Extensive experiments demonstrate
121 the effectiveness of our method against state-of-the-
122 art methods, especially in top-1 precision.

123 2 RELATED WORK

124 Many fields can benefit from network alignment, such
125 as user anonymous identification in social networks [11],
126 comparing schemas between databases [25], linking entities
127 among multiple knowledge graphs [26] and aligning proteins
128 between species [27]. Research on static network
129 alignment has seen much development, and numerous
130 approaches have been proposed. In the past two years,
131 researchers have also begun to focus on dynamic network
132 alignment. According to different scenarios, we introduce
133 existing network alignment methods in two parts: static
134 network alignment and dynamic network alignment.

135 **Static network alignment** is mostly based on learning
136 network structure and node attributes to judge whether two
137 nodes represent the same entity. Recent network alignment
138 methods mostly use network embedding [28], [29] since
139 that can map the network structure to a low-dimensional
140 space and is beneficial to maintain network structure and
141 learned node representation could be used for comparing
142 the similarity between nodes [13], [14], [15], [16], [17], [18],
143 [19], [24], [30], [31], [32], [33], [34], [35], [36]. According to
144 whether the two networks are merged into one network
145 through known anchor nodes for representation learning,
146 the existing methods can be roughly divided into the fol-
147 lowing two categories:

148 (1) The first category of alignment methods learns node
149 embeddings in different networks respectively, then
150 construct constraints [13], [14], [18], [35] or utilize adversarial
151 learning [17], [19], [31], [34], [36] to make the embedding
152 of the anchor nodes or the distributions of the two net-
153 works similar. The former is generally supervised utilizing
154 known anchor nodes as constraint information. The latter
155 is generally unsupervised, and most utilize generative ad-
156 versarial networks to approximate the distribution of the
157 two networks. For example, GINA [14] utilizes two different
158 encoders to learn reliable spatial features of networks firstly,
159 then uses anchor nodes to constrain learned node rep-
160 resentations for following anchor link prediction. DANA [31]
161 learns node embeddings via maximizing the posterior prob-
162 ability distribution of anchor nodes which is based on the
163 parameter space of graph convolutional networks.

164 (2) The second category of alignment methods utilizes
165 anchor nodes forming a unified space and then learns their
166 embeddings for alignment [15], [16], [24], [32], [33]. For
167 example, DHNA [32] learns node embeddings of different

168 networks by a variational autoencoder in the same embed-
169 ding space and uses a dual constraint mechanism to balance
170 the consistency and heterogeneity in network alignment.
171 BRIGHT [33] uses the one-hot vectors of anchor links to
172 form the bases of common embedding space, and other
173 initial embeddings of non-anchor nodes are obtained by a
174 random walk with the restart. Then it uses a shared linear
175 layer to train the weights of scores from different anchor
176 links by keeping node embeddings of different networks in
177 the same embedding space.

178 Although these static network alignment methods have
179 demonstrated good performance, ignoring the temporal in-
180 formation of the network makes them unable to accurately
181 model real-world scenarios to achieve more accurate per-
182 formance, usually having poor top-1 precision.

183 **Dynamic network alignment** focuses more on how
184 to use time information to improve the alignment effect
185 in contrast to static network alignment. DNA [37] uses
186 an LSTM encoder to learn evolvement neighborhood of
187 nodes, and puts a consistency regularization onto the heart
188 of the LSTM to keep the representation similarity with
189 the neighbors of the node. DGA [38] expands based on
190 DNA that uses an attentive graph convolution to model
191 the structural information of nodes and the LSTM unit to
192 incorporate the temporal evolvement pattern of nodes in
193 the dynamic network. Unlike DNA and DGA which focus
194 on the evolution of the entire network, HDyNA [39] only
195 focuses on newly emerging nodes in the network. As a new
196 node is added, its weights are learned heuristically, and then
197 second-order proximity is preserved in updating the local
198 network. CTSA [40] aligns the same entity across different
199 snapshots in one dynamic network, which differs from our
200 work that aligns snapshots in two different networks.

201 Compared with existing methods, our method not only
202 focuses on the evolution of the local structure of nodes
203 in the network, but also pays attention to the global-local
204 evolution patterns of anchor node pairs and keeps its consis-
205 tency changing over time, which is important for predicting
206 potential anchor links.

207 3 FORMAL DEFINITION

208 Referring to Fig.1(a), in dynamic scenarios, we focus solely
209 on the structure of a time-stamped network. We partition the
210 network into slices, as depicted in Fig.1(a), and construct
211 a series of snapshots in the time domain. Each snapshot
212 represents the network's characteristics in the correspond-
213 ing time slice. Thus, a network with time-stamped can be
214 defined as $G = \{G^1, G^2, \dots, G^t, \dots, G^T\}$. T is the number
215 of snapshots. Each $G^t = (V^t, E^t)$ is an undirected and
216 unweighted network snapshot at time t . V^t is the set of
217 nodes and E^t is the set of edges at t . Considering that
218 we align two time-stamped networks, we use superscripts
219 to distinguish them, i.e., $G^1 = \{G^{1,1}, G^{1,2}, \dots, G^{1,T}\}$ and
220 $G^2 = \{G^{2,1}, G^{2,2}, \dots, G^{2,T}\}$. In general, these two net-
221 works are partially overlapped by anchor nodes, which ap-
222 pear in both G^1 and G^2 . Anchor nodes of network snapshot
223 t are stored in set $\mathcal{A}^t = \{(v_i^{1,t}, v_j^{2,t})\}$. Each element in
224 \mathcal{A}^t represents an anchor link, i.e., the element $(v_i^{1,t}, v_j^{2,t})$
225 describes $v_i^{1,t}$ in $G^{1,t}$ corresponds to $v_j^{2,t}$ in $G^{2,t}$.

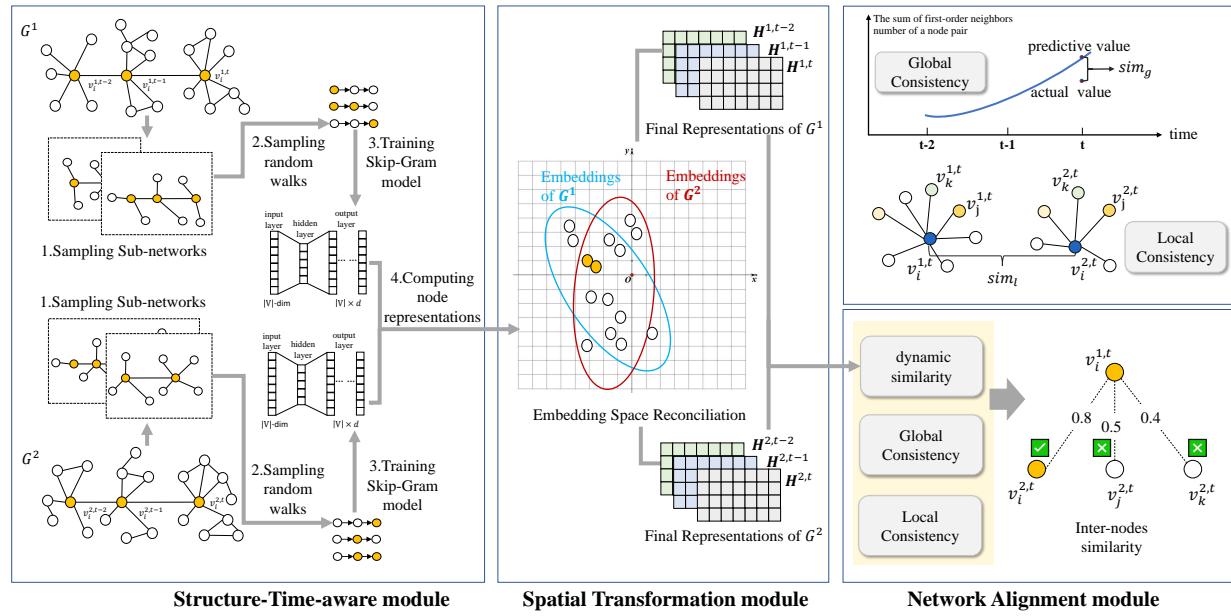


Fig. 3: The overview architecture of GLDyNA: For each dynamic network, the snapshots are sampled and trained to learn node representations. Then, the node representations of G^1 are reconciled with the embedding space of G^2 's node representations. Finally, we perform network alignment based on dynamic similarity and Global-Local Consistency.

Dynamic Network Alignment. Given two partially overlapped networks G^1 and G^2 with anchor link set $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \dots, \mathcal{A}^T\}$, the dynamic network alignment task is to find all potential anchor links in each network snapshot, i.e., we aim to learn a predictive function for each network snapshot: $f^t : (G^{1,t}, G^{2,t}, \mathcal{A}^t) \rightarrow Y$, Y_{ij}^t represents the probability that $v_i^{1,t}$ and $v_j^{2,t}$ are predicted to be a pair of potential anchor nodes across networks $G^{1,t}$ and $G^{2,t}$.

4 METHODOLOGY

In this section, we introduce the details of the proposed GLDyNA. As shown in Fig. 3, GLDyNA includes three modules, i.e., **Structure-Time-aware module**, **Spatial Transformation module**, and **Network Alignment module**. Through the Structure-Time-aware module, we first learn the node representations of the two networks separately so that we can capture information about the network structure and the temporal information for the following alignment. Then, we utilize a Spatial Transformation module to map the node representations of G^1 into the node representation space of G^2 . The Network Alignment module performs network alignment based on the inter-nodes similarity measures, which include dynamic similarity and global-local consistency of nodes.

4.1 Structure-Time-aware module

In the Structure-Time-aware module, we design a method based on random walks to encode node representations that consider the changes in node behavior over time. As shown in Fig. 3, the behavior of a node (e.g., $v_i^{1,t}$) can be influenced by its nearby nodes in the current snapshot network as well as the preceding self-nodes (e.g., $v_i^{1,t-1}$ and $v_i^{1,t-2}$) and their first-level neighbors in previous snapshot networks.

Therefore, we conduct the following steps to sample Spatial-Temporal sub-networks starting from the given node to learn the node representations:

For a given node $v_i^{1,t}$ in snapshot $G^{1,t}$, we consider the influence of the nodes in the current snapshot and the nodes of the previous τ historical snapshots on it. For each snapshot $G^{1,k}$, $k \in \{0, 1, \dots, \tau\}$, we sample the first-order neighborhood nodes of $v_i^{1,t-k}$ and denote the sampled sub-network as $G_{sample}^{1,k}$. Then we merge $G^{1,t}$, all $v_i^{1,t-k}$ and all $G_{sample}^{1,k}$ to get the Spatial-Temporal sub-network $G_{ST}^{1,k}$ of $v_i^{1,t}$.

After obtaining the Spatial-Temporal sub-network $G_{ST}^{1,k}$ of $v_i^{1,t}$, the representations of nodes in $G_{ST}^{1,k}$ are learned by performing random walks on $G_{ST}^{1,k}$. Specifically, for each given node $v_i^{1,t}$ we generate a random walk sequence of length l and denote the sequence as $\mathcal{W}_{v_i^{1,t}}$. When two nodes have many edges or neighbors, they will be visited more frequently during random walks, showing that their network structures are similar. As a result, representations of these two nodes in the embedding space should be close to each other. Given a random walk sequence $\mathcal{W}_{v_i^{1,t}}$, it is now possible to formulate learning spatial-temporal-preserving node embeddings as an optimization problem as follows:

$$\max_{\mathbf{h}} \sum_{v_j \in \mathcal{W}_{v_i^{1,t}}} \log \Pr(\mathcal{W}_w = \{v_{j-w}, \dots, v_{j+w}\} \setminus v_j \mid \mathbf{h}(v_j)), \quad (1)$$

where $\mathbf{h} : V \rightarrow \mathcal{R}^d$ is the embedding function that maps a given node to a d -dimensional representation, $\mathbf{h}(v_j)$ is the representation of v_j . w is the context window size for optimization. We assume conditional independence between the nodes of a context window when observed with respect

285 to the source node v_j :

$$\begin{aligned} \Pr(\mathcal{W}_w = \{v_{j-w}, \dots, v_{j+w}\} \setminus v_j \mid \mathbf{h}(v_j)) \\ = \prod_{v_k \in \mathcal{W}_w} \Pr(v_k \mid \mathbf{h}(v_j)). \end{aligned} \quad (2)$$

286 The probability could be calculated as follows:

$$\Pr(v_k \mid \mathbf{h}(v_j)) = \frac{\exp(\mathbf{h}(v_k) \mathbf{h}(v_j))}{\sum_{v_n \in \mathcal{N}(v_j)} \exp(\mathbf{h}(v_n) \mathbf{h}(v_j))}, \quad (3)$$

287 where $\mathcal{N}(v_j)$ represents the neighbors set of node v_j . To
288 learn such a representation that captures the relationship of
289 a node with other co-occurring nodes in a window, we use
290 a similar Skip-Gram algorithm as proposed in [41] to learn
291 the node representations from random walks of a network.

292 Therefore, through Eq(1) which maximizes the co-
293 occurrence of neighborhood nodes at snapshot t and in
294 previous $(t - \tau)$ snapshots together, the Structure-Time-
295 aware module learns node representations in G^1 and G^2
296 respectively. We denote \mathbf{h}^1 as the representations of nodes
297 in G^1 , and \mathbf{h}^2 as the representations of nodes in G^2 .

298 4.2 Spatial Transformation module

299 Across different networks, the same nodes may exhibit
300 varying characteristics and interaction relationships due to
301 semantic distinctions of networks. As a consequence, the
302 embedding spaces of node representations differ, making it
303 infeasible to directly utilize the learned representations for
304 similarity measurement of nodes and network alignment.
305 Therefore, we propose a method to reconcile the embedding
306 spaces of networks G^1 and G^2 . To accomplish nonlinear
307 spatial transformations, we employ a feed-forward neural
308 network to map node representations from G^1 to the em-
309 bedding space of node representations in G^2 :

$$\phi(\mathbf{h} | \mathbf{W}_1, \mathbf{W}_2, \mathbf{b}) = \sigma(\mathbf{h} \mathbf{W}_1 + \mathbf{b}) \mathbf{W}_2, \quad (4)$$

310 where $\sigma(\cdot)$ is an activation function and we use the Sigmoid
311 function in this paper. \mathbf{W}_1 , \mathbf{W}_2 , and \mathbf{b} are trainable par-
312 ameters.

313 In the reconciled embedding space, the representations
314 of the same entity (i.e., anchor nodes) should be as similar
315 as possible, and even be the same ideally. Therefore, we
316 use labeled anchor links to constrain the representation
317 of anchor nodes and conduct the training of the above-
318 mentioned feed-forward neural network to obtain the ideal
319 spatial transformation function $\phi(\cdot)$:

$$\mathcal{O}_\phi = \sum o_i, \quad (5)$$

$$o_i = \begin{cases} 1 - \theta(\phi(\mathbf{h}^1(v_i^1)), \mathbf{h}^2(v_j^2)) & (v_i^1, v_j^2) \in \mathcal{A} \\ \max(0, \theta(\phi(\mathbf{h}^1(v_i^1)), \mathbf{h}^2(v_j^2)) - \varepsilon) & (v_i^1, v_j^2) \notin \mathcal{A}, \end{cases} \quad (6)$$

320 where $\theta(\cdot, \cdot)$ is the cosine value of two node representations,
321 ε is a hyperparameter, and we adopt SGD [42] to minimize
322 Eq(5). In this way, the representations of the corresponding
323 anchor nodes are almost identical. Meanwhile, the represen-
324 tations of other non-anchor nodes in the two networks can
325 maintain their own structural features.

4.3 Network Alignment module

327 The Network Alignment module includes three parts:

328 **Dynamic similarity.** With the node representation
329 $\phi(\mathbf{h}^1)$ and \mathbf{h}^2 that in the same embedding space, a direct
330 way of determining the alignments for a node is to calculate
331 pairings of similarity between the representations that
332 contain dynamics, i.e., nodes dynamic similarity:

$$sim_\theta(v_i^{1,t}, v_j^{2,t}) = \theta(\phi(\mathbf{h}^1(v_i^1)), \mathbf{h}^2(v_j^2)). \quad (7)$$

334 As we introduced in Section 4.1, the Structure-Time-aware
335 module focuses on the dynamic evolution of nodes in one
336 network over time, sim_θ reflects the evolution similarity
337 of two nodes in their respective networks. The larger the
338 sim_θ value, the higher the probability that v_i^1 and v_j^2 are
339 corresponding anchor nodes.

340 When aligning nodes between different networks, in
341 addition to considering the evolution process of nodes in
342 different networks, we also consider patterns (global and
343 local) between node pairs over time.

344 **Global consistency.** At the global level, for the dynamic
345 network, its scale usually evolves with obvious distributions
346 over time at the global level, such as a sigmoid curve [43]
347 or a power-law distribution [44]. For each anchor node pair,
348 there also exists a similar pattern. As shown in Fig. 2, when
349 the network evolves, the first-order neighbors number c_i^1
350 and c_i^2 of an anchor node pair (v_i^1, v_i^2) vary over time in
351 different networks respectively, and their sum (i.e., $c_i^1 + c_i^2$)
352 obeys a certain underlying pattern, such as linear increasing.
353 Such a global evolutionary pattern is common in real net-
354 work alignment scenarios. For example, a scholar who col-
355 laborates with other scholars to publish papers in journals
356 and conferences, respectively, will have different propensities
357 in the journal and conference collaboration over time,
358 but the total number of people he keeps collaborating with
359 generally remains the same or even increases. Hence, we
360 use a linear neural network to learn the sum collaboration
361 (i.e., first-order neighbors number) of anchor node pairs at
362 snapshot t as shown in the top rightmost side of Fig. 3:

$$\psi(c_p^t | \mathbf{W}_g, \mathbf{b}) = \mathbf{c} \mathbf{W}_g + \mathbf{b}, \quad (8)$$

363 where $\mathbf{c} = \{c^0, c^1, \dots, c^{t-1}\}$ is a vector composed of the
364 sum of first-order neighbors number of an anchor node pair
365 before snapshot t . \mathbf{W}_g and \mathbf{b} are trainable parameters. c_p^t
366 is the predicted sum first-order neighbors number of an
367 anchor node pair at snapshot t .

368 Thus, for an arbitrary node pair (v_i^1, v_j^2) , we use the
369 difference between the predicted sum first-order neighbors
370 number $c_p^t(v_i^1, v_j^2)$ and the true sum first-order neighbors
371 number $c^t(v_i^1, v_j^2)$ at snapshot t as the global consistency
372 measure to determine the probability that two nodes are the
373 corresponding anchor nodes at snapshot t :

$$sim_g(v_i^{1,t}, v_j^{2,t}) = \exp^{-|c_p^t(v_i^1, v_j^2) - c^t(v_i^1, v_j^2)|}. \quad (9)$$

374 The larger the sim_g value, the higher the probability that v_i^1
375 and v_j^2 are corresponding anchor nodes.

376 **Local consistency.** At the local level, nodes in net-
377 works usually exist homophily equivalence [45]. Homophily
378 equivalence refers to the phenomenon in which adjacent
379 nodes in a network display similar characteristics or at-
380 tributes. This concept bears resemblance to the main idea

TABLE 1: Statistics of the datasets.

Dataset		Networks						
		Twitter		Foursquare		Anchor Links		
Snapshots	# nodes	# edges	# nodes	# edges	# total	# added	# disappeared	
TF	t_0	4,709	107,528	4,836	54,586	596	-	-
	t_1	4,809	124,585	4,936	60,330	854	440	182
	t_2	4,909	140,338	5,036	67,044	987	312	179
	t_3	5,009	152,434	5,136	72,732	1,078	207	116
	t_4	5,109	164,936	5,236	76,874	1,282	204	0
	journal-paper cooperation		conference-paper cooperation		Anchor Links			
JC	Snapshots	# nodes	# edges	# nodes	# edges	# total	# added	# disappeared
	t_0	2,832	11,164	4,343	15,354	1,013	-	-
	t_1	2,997	11,900	4,755	24,055	1,036	570	540
	t_2	3,259	11,911	5,203	26,216	1,141	684	579
	t_3	3,708	12,658	6,333	44,099	1,352	827	616
	t_4	4,153	16,467	6,343	48,965	1,359	741	734
	t_5	4,687	20,851	6,573	41,612	1,401	773	731
	t_6	5,885	27,200	7,134	41,580	1,799	1,099	701

381 behind the Word2Vec method in natural language processing,
382 where words that frequently co-occur are likely to possess similar meanings or representations. As shown in
383 the top rightmost side of Fig. 3, according to the homophily
384 equivalence hypothesis, nodes $v_i^{1,t}$, $v_j^{1,t}$, and $v_k^{1,t}$ exhibit
385 greater similarity in the embedding space of $G^{1,t}$, and the
386 same for $v_i^{2,t}$, $v_j^{2,t}$, and $v_k^{2,t}$ in the embedding space of
387 $G^{2,t}$. The existence of significant similarities among nodes
388 $v_i^{1,t}$, $v_j^{1,t}$, and $v_k^{1,t}$ in a network can result in confusion
389 when comparing node $v_i^{2,t}$ with node $v_i^{1,t}$ by calculating
390 nodes dynamic similarity sim_θ . Furthermore, this can lead
391 to incorrect alignment of node $v_i^{2,t}$ with nodes $v_j^{1,t}$ or $v_k^{1,t}$.
392 Inspired by the principle of structural equivalence [46],
393 i.e., if two nodes share many common neighbors in the
394 network, then they are structural equivalence, we define
395 the following measurement to calculate the local similarity
396 between two nodes across networks to alleviate the impact
397 of node homophily:
398

$$sim_l(v_i^{1,t}, v_j^{2,t}) = \sum_{\varsigma}^t \frac{\log(|\mathcal{N}(v_i^{1,t}) \cap \mathcal{N}(v_j^{2,t})| + 1)}{(t - \varsigma + 1)}, \quad (10)$$

399 where $\mathcal{N}(v_i^{1,t})$ and $\mathcal{N}(v_j^{2,t})$ denote the set of all the
400 first-order neighbour of node $v_i^{1,t}$ and $v_j^{2,t}$, respectively.
401 $|\mathcal{N}(v_i^{1,t}) \cap \mathcal{N}(v_j^{2,t})|$ is the number of known pairs of anchor
402 nodes existing in their neighborhood. ς is the hyperparameter
403 that determines the number of previous snapshots, that
404 GLDyNA considers for up to $(t - \varsigma + 1)$ local neighborhood
405 information when enforcing local consistency constraints.
406 The larger the sim_l value, the higher the probability that v_i^1
407 and v_j^2 are corresponding anchor nodes.

408 Finally, we compare the similarity of cross-network
409 nodes based on the aforementioned introduced dynamic
410 similarity and global-local consistency of nodes:

$$sim(v_i^{1,t}, v_j^{2,t}) = sim_\theta(v_i^{1,t}, v_j^{2,t}) + \lambda * sim_g(v_i^{1,t}, v_j^{2,t}) + \gamma * sim_l(v_i^{1,t}, v_j^{2,t}), \quad (11)$$

411 where λ and γ are the weight of global consistency and
412 local consistency respectively. Based on $sim(\cdot, \cdot)$, we obtain

pairwise similarities Y_{ij}^t between nodes to be aligned in
413 two networks and sort them according to their similarity
414 scores. Since not all nodes have corresponding nodes in the
415 other network, we set a threshold ε . When $sim(\cdot, \cdot) \geq \varepsilon$, we
416 consider the node pair with the highest $sim(\cdot, \cdot)$ value as a
417 potential anchor node pair.
418

4.4 Time Complexity

420 The time complexity of GLDyNA primarily lies in the node
421 representation learning process. GLDyNA involves sam-
422 pling and generating Spatial-Temporal sub-networks for
423 each node, which has a time complexity of $O(NM)$. Here,
424 N represents the total number of nodes across all snapshots
425 in the network, and M denotes the average number of
426 first-order neighbors for these nodes. Additionally, conduct-
427 ing random walks on the generated Spatial-Temporal sub-
428 networks incurs a time complexity of $O(nl)$, where l is
429 the length of the random walk, and n is the number of
430 walk iterations. Moreover, the time complexity of model
431 training based on the obtained sequences is $O(m)$, where
432 m represents the number of training iterations. Therefore,
433 the overall time complexity of GLDyNA can be expressed
434 as $O(NM + nl + m) = O(NM)$.

5 EXPERIMENTS

435 This section introduces the datasets and describes the set of
436 experiments conducted to validate the proposed GLDyNA.
437 Additionally, we analyze the validity of each component of
438 the model and the influence of model parameters.
439

5.1 Experimental Settings

5.1.1 Datasets

440 The following two datasets are used to verify the effective-
441 ness of GLDyNA.
442

- 443 • Social Networks: This data set includes users and
444 their followers from Twitter and Foursquare (TF),
445 respectively. The snapshots are at equal intervals
446

447 of the network, and there exist new nodes in each
448 snapshot [39].

- 449 • Academic Networks: This data set includes re-
450 searchers and their collaborators from DBLP. De-
451 pending on the publication channels of researchers'
452 papers, academic networks are divided into journal-
453 paper cooperation networks and conference-paper
454 cooperation networks (JC). And each snapshot rep-
455 presents a year.

456 Details are illustrated in Table 1. We have expanded
457 based on the above two datasets, referred to as TF+ and JC+.
458 The extension rule is that except for the snapshot network at
459 t_0 , all other snapshot networks are merged by themselves,
460 along with all snapshots at all previous times.

461 5.1.2 Baseline Methods

462 The proposed method is compared with the seven state-of-
463 the-art methods listed below.

- 464 • BRIGHT [33]: a static network alignment method that
465 creates a space by RWR whose bases are anchor node
466 encoding vectors, followed by a shared linear layer
467 to learn node representations.
- 468 • NetTrans [30]: a static network alignment method
469 that uses graph convolutional network to learn node
470 representations at different resolutions for alignment
471 from the network transformation view.
- 472 • DANA [31]: a static network alignment method that
473 uses GCN to learn node embeddings and train an
474 adversarial domain classifier supervised by the an-
475 chor nodes to obtain domain-invariant features for
476 alignment.
- 477 • NeXtAlign [24]: a static network alignment method
478 that uses a special graph convolutional network to
479 balance the consistency and disparity in alignment
480 through the learning process.
- 481 • DHNA [32]: a static network alignment method that
482 uses a variational autoencoder to learn node em-
483 beddings, and considers the different anchor nodes'
484 degrees across networks.
- 485 • DGA [38]: a dynamic network alignment method
486 that uses a dynamic graph autoencoder to learn
487 user embeddings in each network, and constructs a
488 common subspace for user alignment across different
489 networks.
- 490 • HDyNA [39]: a dynamic network alignment method
491 that learns the local influence weight of new nodes
492 in a single network environment using an attention
493 mechanism and anchor nodes are used as supervised
494 information.

495 5.1.3 Evaluation Metrics

496 For each matching pair $(v_i^{1,t}, v_j^{2,t})$ in the test set, we rank
497 the target nodes in the result according to Y_{ij}^t . To quantify
498 the ranking at snapshot t , we use the two evaluation metrics
499 which are commonly used in network alignment tasks.

- 500 • $Precision = \frac{|M_t @ 1|}{|U_t|}$ indicates whether the true positive
501 match occurs in top-1 candidates, where $|M_t @ 1|$
502 is the count of the correct alignments between net-
503 works $G^{1,t}$ and $G^{2,t}$ in top-1 choices, and $|U_t|$ is the
504 number of anchor links in the train set.

- 505 • $MRR = \frac{1}{|U_t|} \sum_{(v_i^{1,t}, v_j^{2,t}) \in \mathcal{T}} \frac{1}{rank(v_j^{2,t})}$, where
506 $rank(v_j^{2,t})$ is the rank of true anchor target in
507 the sorted list of anchor candidates. \mathcal{T} is the test set
508 that includes correct alignments between $G^{1,t}$ and
509 $G^{2,t}$.

510 5.1.4 Implementation Details

511 To create our training and testing datasets, we randomly
512 partitioned the anchor nodes into two sets. The ratio of
513 the number of anchor nodes in the training set to that in
514 the testing set was 4:1, with the specific numbers randomly
515 sampled. For a fair comparison, hyper-parameters except for
516 node embedding dimension are set to default for all base-
517 lines. We set the hyper-parameters of GLDyNA as follows
518 unless otherwise specified:

- 519 - For the Structure-Time-aware module, We set the
520 number of historical snapshots considered during
521 sampling $\tau = 1$, and the random walk length $l = 15$,
522 the node representation dimension $d = 64$.
- 523 - For the Network Alignment module, we set the $\varsigma = 0$
524 in local consistency measurement, i.e., we consider
525 the local consistency of all previous snapshots. We set
526 the weight of global consistency $\lambda = 0.15$ and are the
527 weight of local consistency $\gamma = 0.1$. The threshold
528 ε for aligning potential anchor node pairs is set as
529 the average similarity of anchor node pairs in the
530 training set.

531 The experimental environment uses Python 3.7 lan-
532 guages as the basic development language, and GLDyNA is
533 implemented based on the open-source Pytorch framework.
534 Experiments are performed on a workstation equipped with
535 NVIDIA RTX 1080Ti 20GB video memory. In each experi-
536 ment, we repeated it 10 times and reported the mean with a
537 95% confidence interval.

538 5.2 Model Performance Analysis

539 **Precision Improvement.** We first compare GLDyNA with
540 all baselines in four datasets, and results are reported in Ta-
541 ble 2 and Table 3. Results show that the proposed GLDyNA
542 mostly outperforms the baselines on both *Precision* and
543 *MRR*. On *Precision*, GLDyNA achieves the best perfor-
544 mance on all snapshot networks of four datasets, improving
545 by at least 9.97%, 4.17% compared with the best competitors
546 on dataset TF (TF+), and JC (JC+) respectively. On *MRR*,
547 GLDyNA achieves the best performance on all snapshot net-
548 works except for t_2 snapshot of JC and JC+, improving by an
549 average 17.63%, 5.65% compared with the best competitors
550 on dataset TF (TF+), and JC (JC+) respectively.

551 Compared with the static network alignment method,
552 GLDyNA has achieved a very significant improvement in
553 alignment accuracy, which demonstrates the effectiveness
554 of considering the evolution characteristics of nodes over
555 time. The suboptimal performance of these static methods
556 on JC and TF datasets suggests that relying solely on one
557 snapshot for alignment may overlook valuable information.
558 When temporal information is disregarded and multiple
559 snapshots are merged, as in the JC+ and TF+ datasets, the
560 performance of static methods does not improve and some

TABLE 2: Experimental results on TF and TF+ datasets at different snapshot t . The best and second-best results are highlighted in boldface and underlined, respectively. $\Delta\%$ denotes the improvement of GLDyNA compared to the best baseline methods results.

		t_1		t_2		t_3		t_4	
		<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>
TF	BRIGHT	0.1111	0.1779	0.1515	0.2036	0.1481	0.2127	0.1167	0.1819
	NetTrans	0.1462	0.1935	0.1263	0.1764	0.1203	0.1710	0.1323	0.1926
	DANA	0.1521	0.2187	0.1313	0.1897	0.1412	0.2188	0.1498	0.2373
	NeXtAlign	0.0702	0.1272	0.0808	0.1321	0.0694	0.1256	0.0739	0.1344
	DHNA	0.1022	0.1100	0.1154	0.1327	0.0912	0.1035	0.1159	0.1677
	DGA	<u>0.6233</u>	<u>0.6419</u>	<u>0.6800</u>	<u>0.6921</u>	<u>0.7301</u>	<u>0.7622</u>	<u>0.8117</u>	<u>0.8337</u>
	HDyNA	0.5246	0.5721	0.6311	0.6871	0.7200	0.7596	0.7657	0.8003
TF+	GLDyNA	0.7045	0.7059	<u>0.8871</u>	<u>0.8872</u>	<u>0.9024</u>	<u>0.9025</u>	0.9500	0.9500
	$\Delta\%$	13.03	9.97	30.46	28.19	23.60	18.41	17.04	13.95
	BRIGHT	0.0936	0.1543	0.1060	0.1527	0.1574	0.2263	0.1712	0.2393
	NetTrans	0.1403	0.1871	0.1212	0.1733	0.1157	0.1684	0.1361	0.1947
	DANA	0.1462	0.2173	0.1288	0.1932	0.1412	0.2221	0.1537	0.2386
	NeXtAlign	0.0819	0.1297	0.0657	0.1285	0.0648	0.1265	0.0895	0.1253
	DHNA	0.1092	0.1156	0.1054	0.1335	0.1029	0.1147	0.1377	0.1691
JC	DGA	<u>0.6233</u>	<u>0.6419</u>	<u>0.6800</u>	<u>0.6921</u>	<u>0.7301</u>	<u>0.7622</u>	<u>0.8117</u>	<u>0.8337</u>
	HDyNA	0.5246	0.5721	0.6311	0.6871	0.7200	0.7596	0.7657	0.8003
	GLDyNA	0.7045	0.7059	<u>0.8871</u>	<u>0.8872</u>	<u>0.9024</u>	<u>0.9025</u>	0.9500	0.9500
	$\Delta\%$	13.03	9.97	30.46	28.19	23.60	18.41	17.04	13.95

TABLE 3: Experimental results on JC and JC+ datasets at different snapshot t . The best and second-best results are highlighted in boldface and underlined, respectively. $\Delta\%$ denotes the improvement of GLDyNA compared to the best baseline methods results.

		t_1		t_2		t_3		t_4		t_5		t_6	
		<i>Precision</i>	<i>MRR</i>										
JC	BRIGHT	0.2836	0.4115	0.2794	0.4247	0.2693	0.4018	0.3014	0.4152	0.2419	0.3898	0.2611	0.3921
	NetTrans	0.1352	0.1818	0.1271	0.1575	0.1037	0.1404	0.0774	0.1185	0.1142	0.1697	0.0612	0.1025
	DANA	0.4589	0.5497	0.4276	0.4691	0.3963	0.4566	0.3856	0.4568	0.3732	0.4468	0.3481	0.4177
	NeXtAlign	0.3188	0.4310	0.2456	0.3514	0.2815	0.3795	0.2583	0.3487	0.2500	0.3833	0.2256	0.2882
	DHNA	0.2907	0.3011	0.3270	0.3609	0.2571	0.2834	0.2279	0.2630	0.2800	0.3107	0.2112	0.2971
	DGA	<u>0.5022</u>	<u>0.5500</u>	<u>0.4729</u>	0.5273	<u>0.5000</u>	<u>0.5388</u>	<u>0.4992</u>	<u>0.5236</u>	<u>0.5235</u>	<u>0.5700</u>	<u>0.6122</u>	<u>0.6503</u>
	HDyNA	0.4304	0.4972	0.4641	0.5210	0.4090	0.5319	0.4175	0.5100	0.5010	0.5541	0.5399	0.6110
JC+	GLDyNA	0.5702	0.5743	<u>0.4926</u>	<u>0.4969</u>	0.6126	0.6070	0.5608	0.5605	<u>0.5974</u>	<u>0.6100</u>	<u>0.7078</u>	0.7057
	$\Delta\%$	13.54	4.42	4.17	-5.77	22.52	12.66	12.34	7.05	14.12	7.02	15.62	8.52
	BRIGHT	0.2711	0.3900	0.2807	0.4166	0.2702	0.4110	0.3158	0.4252	0.2700	0.3961	0.2600	0.3851
	NetTrans	0.1357	0.1899	0.1193	0.1496	0.1201	0.1370	0.1022	0.1257	0.1100	0.1636	0.0895	0.1103
	DANA	0.4402	0.5152	0.4270	0.4672	0.4117	0.4983	0.4000	0.4794	0.4457	0.5029	0.3665	0.4570
	NeXtAlign	0.2901	0.3510	0.2500	0.3766	0.3011	0.4067	0.2594	0.3499	0.3143	0.4402	0.3147	0.3800
	DHNA	0.2801	0.2900	0.3206	0.3551	0.2764	0.3004	0.2300	0.2719	0.2807	0.3233	0.2410	0.3306
JC+	DGA	<u>0.5022</u>	<u>0.5500</u>	<u>0.4729</u>	0.5273	<u>0.5000</u>	<u>0.5388</u>	<u>0.4992</u>	<u>0.5236</u>	<u>0.5235</u>	<u>0.5700</u>	<u>0.6122</u>	<u>0.6503</u>
	HDyNA	0.4304	0.4972	0.4641	0.5210	0.4090	0.5319	0.4175	0.5100	0.5010	0.5541	0.5399	0.6110
	GLDyNA	0.5702	0.5743	<u>0.4926</u>	<u>0.4969</u>	0.6126	0.6070	0.5608	<u>0.5974</u>	<u>0.6100</u>	<u>0.7078</u>	0.7057	
	$\Delta\%$	13.54	4.42	4.17	-5.77	22.52	12.66	12.34	7.05	14.12	7.02	15.62	8.52

561 even degrade. This highlights the detrimental impact of
562 disregarding the evolution of the nodes on alignment, as
563 confusing information may introduce additional confusion
564 into the alignment process, as discussed in Section 1.

565 Compared to the dynamic network alignment method
566 HDyNA and DGA, GLDyNA still demonstrates better align-
567 ment performance. HDyNA solely considers the scenario
568 where new nodes are added to the evolving network over
569 time while disregarding the situation where certain nodes
570 may also vanish over time. As a result, it fails to roundly
571 capture the temporal dynamics of node features and conse-
572 quently impairs the accuracy of network alignment. DGA
573 utilizes graph attention convolutional units and an LSTM-
574 based encoder to learn representations that capture the
575 dynamic information within nodes from two networks in-
576 dividually. It then aligns the embedded spaces of the two
577 networks by mapping them to a shared subspace. The mech-
578 anism employed for learning node representations requires

579 aligned nodes to exhibit similar neighbor evolution char-
580 acteristics. However, the effectiveness of DGA diminishes
581 when nodes demonstrate divergent evolutionary behaviors.
582 In such scenarios, our method alleviates the challenges
583 posed by limited local consistency in alignment by incor-
584 porating global consistency, thereby maintaining a favor-
585 able alignment performance. While GLDyNA has achieved
586 promising *Precision* and *MRR* alignment results overall,
587 it falls slightly behind DGA in certain snapshot networks.
588 This could be attributed to an imbalanced distribution of
589 anchor nodes in those specific snapshots, where the global
590 consistency negatively affects the alignment performance in
591 those cases.

592 In addition to evaluating the top-1 accuracy of alignment
593 results, we also compared the alignment accuracy of each
594 method at top- α ($\alpha > 1$) levels. The results, as shown
595 in Fig. 4, indicate that as α increases, the accuracy of
596 all comparative methods improves. However, our method

TABLE 4: Experimental results on TF and JC datasets with different anchor node percentage at last snapshot of each dataset (i.e., snapshot 4 of TF and snapshot 6 of JC).

	0.5		0.6		0.7		0.8		
	Precision	MRR	Precision	MRR	Precision	MRR	Precision	MRR	
TF	BRIGHT	0.0874	0.1498	0.1150	0.1816	0.1351	0.1966	0.1167	0.1819
	NetTrans	0.1310	0.1853	0.1306	0.1931	0.1377	0.1926	0.1323	0.1926
	DANA	0.0926	0.1504	0.1131	0.1799	0.1286	0.1980	0.1498	0.2373
	NeXtAlign	0.0562	0.1180	0.0897	0.1282	0.0857	0.1196	0.0739	0.1344
	DHNA	0.0925	0.1137	0.1009	0.1286	0.1143	0.1602	0.1159	0.1677
	DGA	0.6800	0.6904	0.7101	0.7400	0.7581	0.7720	0.8117	0.8337
	HDyNA	0.6533	0.6894	0.7003	0.7220	0.7129	0.7466	0.7257	0.7503
JC	GLDyNA	0.8431	0.8431	0.7901	0.7901	0.8525	0.8525	0.9500	0.9500
	BRIGHT	0.1722	0.2805	0.1875	0.3021	0.2241	0.3364	0.2611	0.3921
	NetTrans	0.0690	0.1049	0.0737	0.1062	0.0761	0.1131	0.0612	0.1025
	DANA	0.3054	0.3296	0.3255	0.3693	0.3516	0.4092	0.3481	0.4177
	NeXtAlign	0.1580	0.2577	0.1892	0.2894	0.2171	0.3036	0.2256	0.2882
	DHNA	0.1997	0.2234	0.2217	0.2534	0.2550	0.2796	0.2112	0.2971
	DGA	0.5733	0.5891	0.5900	0.6092	0.6205	0.6331	0.6122	0.6503
JC	HDyNA	0.5022	0.5571	0.5430	0.5756	0.5402	0.6018	0.5399	0.6110
	GLDyNA	0.6266	0.6259	0.6583	0.6783	0.6778	0.6783	0.7078	0.7057

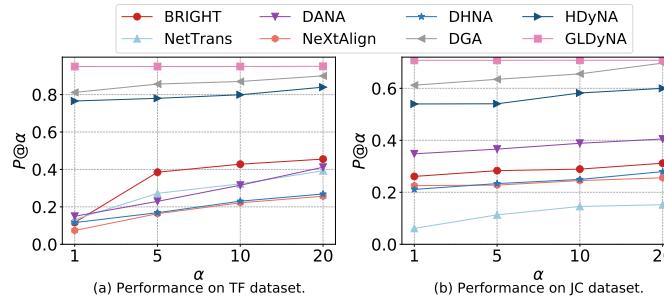


Fig. 4: Experimental results on TF and JC datasets with different top- α metrics at last snapshot of each dataset (i.e., snapshot 4 of TF and snapshot 6 of JC).

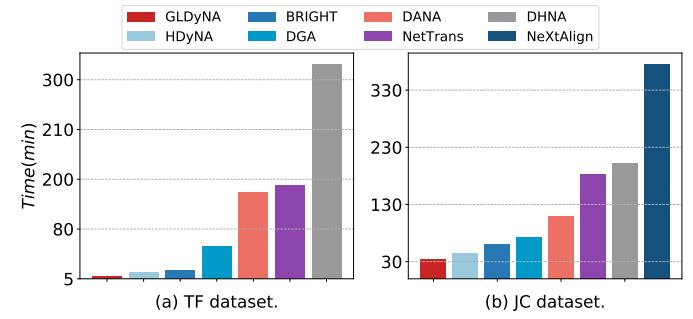


Fig. 5: Model running times on TF and JC datasets.

597 consistently maintains a stable performance. This observation
598 suggests that our approach effectively distinguishes
599 correctly aligned nodes from others, whereas other methods
600 struggle to make clear differentiations, resulting in lower
601 top-1 accuracy but relatively better top- α ($\alpha > 1$) accuracy.

602 **Effect of Anchor Node Percentage.** Based on previous
603 methods, the more anchor nodes in the training set, the
604 better the network alignment performance. We analyze the
605 impact of anchor node percentage in the training set from
606 0.5 to 0.8. As shown in Table 4, the effectiveness of all meth-
607 ods increases with an increasing proportion of anchor nodes
608 in the training set. We observe that GLDyNA outperforms
609 other methods even when the proportion of anchor nodes in
610 the training set is low. This result is due to its considera-
611 tion of dynamic changes in node behaviors, which increases the
612 separability of candidate node pairs, and its incorporation
613 of global consistency, which excludes candidate node pairs
614 that do not conform to the overall evolutionary pattern.

615 **Time for Searching Anchor Node Pairs.** In Fig. 5,
616 we compare the computational efficiency of each method.
617 The running time of most methods is comparable, except
618 for NeXtAlign. Despite NeXtAlign achieving satisfactory
619 alignment results, it utilizes a complex negative sampling
620 method to calculate node attention, resulting in a longer
621 running time. Compared to static methods, although GL-
622 DyNA considers information from different snapshots, it

reduces its running time through a sampling approach.

5.3 Ablation Study

In this subsection, we conduct ablation studies to validate the effectiveness of global-level and local-level consistency. Six variants are designed:

- GLDyNA-WG does not consider the global consistency of node pairs, i.e., performs alignment without $sim_g(\cdot, \cdot)$.
- GLDyNA-WG(L-) does not consider global consistency of node pairs and only considers local consistency between the current snapshot and the previous snapshot, i.e., performs alignment without $sim_g(\cdot, \cdot)$ and set $\varsigma = t - 1$ in $sim_l(\cdot, \cdot)$.
- GLDyNA-WL does not consider local consistency of node pairs, i.e., performs alignment without $sim_l(\cdot, \cdot)$.
- GLDyNA-WGL does not consider global and local consistency of node pairs, i.e., performs network alignment without $sim_g(\cdot, \cdot)$ and $sim_l(\cdot, \cdot)$.
- GLDyNA-WS does not perform spatial transformation, i.e., uses the node representations learned by Structure-Time-aware module directly.
- GLDyNA-G uses a nonlinear neural network to learn the sum collaboration of anchor node pairs as a replacement for Eq(8).

TABLE 5: Results of ablation study on TF dataset at different snapshot t .

	t_1		t_2		t_3		t_4	
	<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>	<i>Precision</i>	<i>MRR</i>
GLDyNA-WG	0.3750	0.3628	0.3549	0.3855	0.3283	0.3244	0.3000	0.2910
GLDyNA-WG(L-)	0.3295	0.3045	0.3871	0.3845	0.2683	0.3193	0.2750	0.2546
GLDyNA-WL	0.6959	0.6906	0.8387	0.8395	0.8926	0.8926	0.9250	0.9250
GLDyNA-WGL	0.2455	0.2658	0.2806	0.2837	0.2732	0.2915	0.3250	0.3303
GLDyNA-WS	0.2219	0.2370	0.2511	0.2466	0.2501	0.2422	0.2991	0.3009
GLDyNA-G	0.6992	0.7059	0.8822	0.8875	0.8906	0.9000	0.9436	0.9461
GLDyNA	0.7045	0.7059	0.8871	0.8872	0.9024	0.9025	0.9500	0.9500

TABLE 6: Results of ablation study on JC dataset at different snapshot t .

	t_1		t_2		t_3		t_4		t_5		t_6	
	<i>Precision</i>	<i>MRR</i>										
GLDyNA-WG	0.5263	0.4827	0.4820	0.4704	0.4364	0.4530	0.3649	0.3825	0.4091	0.3986	0.4373	0.4293
GLDyNA-WG(L-)	0.5251	0.5206	0.4706	0.4470	0.4606	0.4483	0.3649	0.4034	0.4156	0.4351	0.3470	0.3460
GLDyNA-WL	0.5263	0.5377	0.4368	0.4398	0.5333	0.5382	0.5149	0.5180	0.5404	0.5379	0.6758	0.6709
GLDyNA-WGL	0.4439	0.4670	0.4515	0.4703	0.4364	0.4562	0.4338	0.4446	0.4519	0.4626	0.4046	0.4144
GLDyNA-WS	0.4011	0.4318	0.4366	0.4419	0.4052	0.4338	0.4216	0.4288	0.4361	0.4423	0.4000	0.3903
GLDyNA-G	0.5701	0.5699	0.4871	0.4799	0.6112	0.5973	0.5603	0.5500	0.5900	0.5927	0.7000	0.6977
GLDyNA	0.5702	0.5743	0.4926	0.4969	0.6126	0.6070	0.5608	0.5605	0.5974	0.6100	0.7078	0.7057

Table 5 and Table 6 compare the different variants of GLDyNA on TF and JC datasets, respectively. Global and local consistency plays a crucial role in network alignment, which is demonstrated by the significant drop in the performance of GLDyNA-WGL. The superior performance of GLDyNA-WGL compared to most static network alignment baseline methods indicates that the consideration of dynamic node behaviors is beneficial for network alignment.

The advantage of our global consistency can be quantified by the reduced performance of GLDyNA-WG. The advantage of our local consistency can be quantified by the reduced performance of GLDyNA-WL. Comparing the results of GLDyNA-WG on two datasets reveals that the impact of global consistency is more significant on the TF dataset compared to the JC dataset. This discrepancy arises due to the reliance of global consistency on changes in the total number of neighbors for nodes in both networks. In the TF dataset, there is a notable increase in the number of edges between different snapshots, resulting in an overall trend of increasing neighbor count for nodes. This trend facilitates the differentiation of nodes using global consistency. Conversely, the JC dataset demonstrates unstable relationships in the changes of edge count across different snapshots, indicating indistinct variations in neighbor count for nodes. This makes it challenging to differentiate nodes using global consistency, resulting in a relatively limited impact of global consistency in this dataset.

Compared to GLDyNA-WG, GLDyNA-WG(L-)'s performance exhibits a slight decline in both *Precision* and *MRR*, indicating that when disregarding global consistency, the consideration of local consistency with a limited number of snapshots cannot effectively constrain the node pairs. The significant reduction in the performance of GLDyNA-WS indicates that, in the absence of spatial transformation, the learned node representations of the two networks exhibit certain differences due to their semantic disparities, rendering them unsuitable for direct alignment. The performance of GLDyNA-G is comparable to that of GLDyNA, indicating that for the dataset used in the exper-

iments, the majority of changes in the number of neighbors for anchor node pairs still adhere to linear patterns. This finding aligns with reality, where both in social networks and academic collaboration networks, most individuals experience gradual and non-disruptive changes in the number of their connections or friends under normal circumstances.

Furthermore, to investigate whether the sum of first-order neighbors is the optimal feature for computing global consistency, we conducted experiments to validate the use of different order neighbor counts as features for global consistency. The experimental results are shown in Fig. 6 and Fig. 7, which indicate that considering the sum of higher-order neighbors does not improve the effectiveness of alignment. The sum of higher-order neighbors of a node no longer solely represents its intrinsic characteristics but rather reflects the characteristics of its neighbors. As a result, they provide limited useful information for alignment and may even introduce interference. This observation further supports the rationale behind our approach of utilizing only the sum of first-order neighbors to compute global consistency.

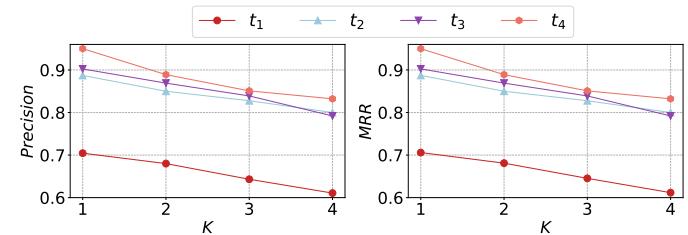


Fig. 6: Experimental results of considering different K-order neighbors in the global consistency on the TF dataset.

5.4 Hyperparameter Sensitivity

To understand the effect of hyperparameters, we analyze accuracy by varying hyperparameters in several experiments. When analyzing each hyperparameter, all other parameters

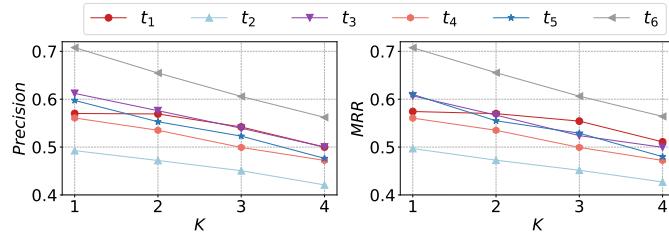


Fig. 7: Experimental results of considering different K-order neighbors in the global consistency on the JC dataset.

712 are held constant at their default values. The results on TF
713 and JC datasets are shown in Fig. 8 and Fig. 9 respectively.

- Impact of the weight of global consistency λ . We examine the impact of varying the global consistency weight across the range of $[0.05, 0.1, 0.15, 0.2]$, and our results reveal that GLDyNA achieves superior performance with $\lambda = 0.15$ in most snapshots. Although anchor node pairs maintain global consistency between them over time, there may be some deviations from global consistency during the evolution process. Therefore, there are some snapshots where $\lambda = 0.15$ does not achieve the best performance.
- Impact of the weight of local consistency γ . We examine the impact of varying the local consistency weight across the range of $[0.05, 0.1, 0.15, 0.2]$, and our results reveal that GLDyNA achieves superior performance with $\gamma = 0.1$. In comparison to global consistency, the performance of $\gamma = 0.1$ across different snapshots is consistently stable, exhibiting negligible occurrences of anchor nodes deviating from local consistency within any given snapshot.
- Impact of the random walk length l . We examine the impact of varying the random walk length across the range of $[5, 10, 15, 20]$, and our results reveal that GLDyNA achieves superior performance with $l = 15$. The walk length l affects the length of the sampled paths and the coverage of the network, thus influencing the learned node representations. A smaller l leads to denser path sampling, capturing local structure better, but may ignore global structure. A larger l can traverse the network more comprehensively but may overlook local dependencies between nodes. Therefore, we choose $l = 15$ considering a balance between the desired representation accuracy and computational efficiency.
- Impact of the dimension of node representations d . We examine the impact of varying the dimension of node representations across the range of $[32, 64, 128, 256]$, and our results reveal that GLDyNA achieves superior performance with 64-dimension node representations. In general, higher node representation dimensions can better preserve the features of nodes in a network. However, in dynamic network alignment tasks, increasing node dimensions may introduce irrelevant information from the evolutionary process, leading to decreased performance.

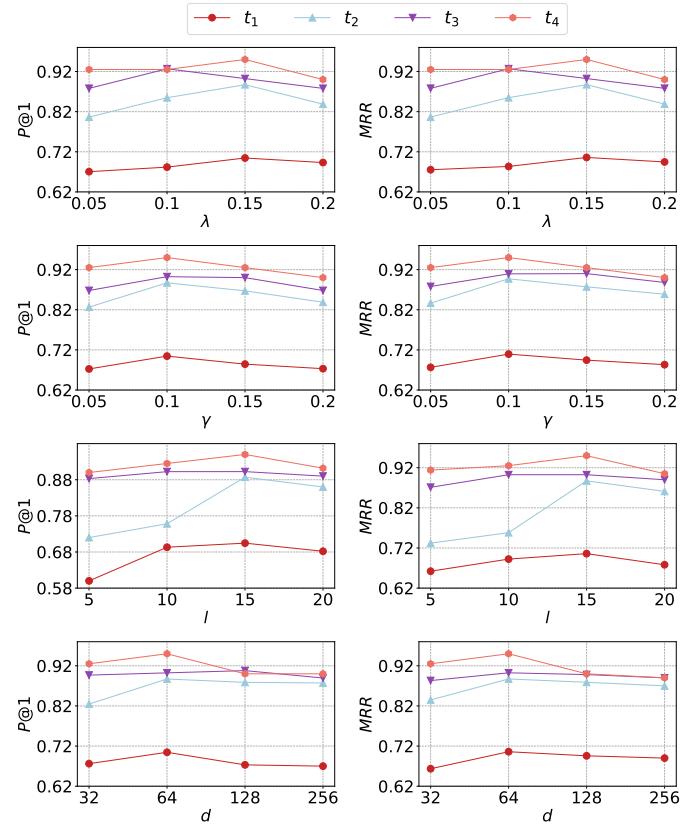


Fig. 8: Experimental results of different model parameters (the weight of global consistency λ , local consistency γ , random walk length l , and node representation dimension d) on the TF dataset.

6 CONCLUSION

This paper mainly investigates the problem of network alignment in dynamic scenarios. The dynamic nature of networks harbors distinctive patterns that can aid in network alignment. To efficiently utilize the dynamics of networks, we propose a method called GLDyNA to improve the accuracy of network alignment. In the proposed GLDyNA, to capture the intra-network dynamics, we design a Structure-Time-aware module to learn the node representations with network dynamics. To address the inter-network alignment, we ensure the consistency of anchor node pairs from global and local views, respectively. Compared to the STOA alignment methods on real-world datasets, GLDyNA can achieve comparable accuracy performance in dynamic scenarios.

In further research, we endeavor to investigate the intrinsic mechanism of the neighborhood structures of a pair of anchor nodes across disparate networks. Specifically, we aim to generate the neighborhood structure of an anchor node in one network based on its neighborhood structure and historical evolution in another network. By examining the intrinsic mechanism, we can gain a deeper understanding of how these structures are formed and how they evolve. It can not only improve the network alignment but also enhance the interpretability of the results.

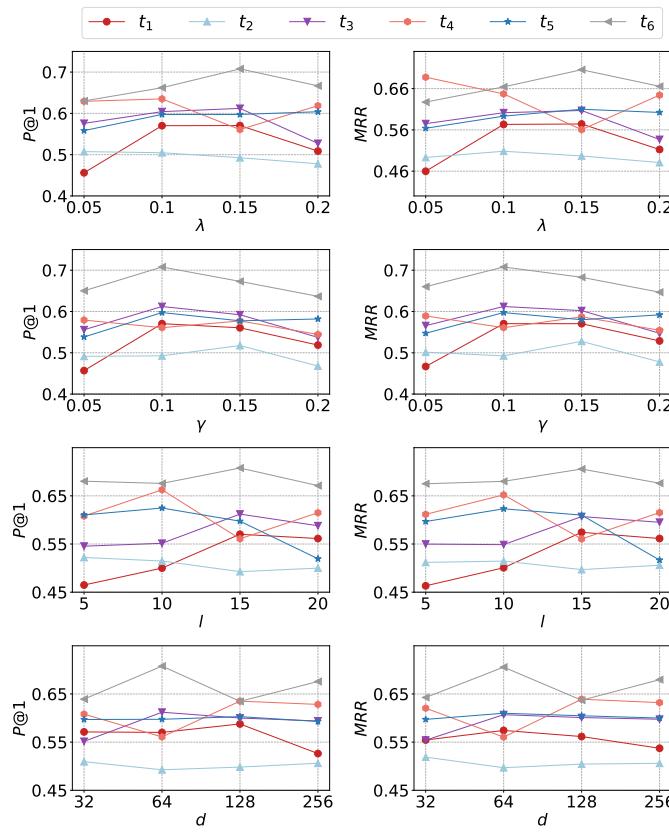


Fig. 9: Experimental results of different model parameters (the weight of global consistency λ , local consistency γ , random walk length l , and node representation dimension d) on the JC dataset.

ACKNOWLEDGMENTS

This work was supported in part by the Sustainable Development Project of Shenzhen (KCXFZ20201221173013036), the Zhejiang Provincial Natural Science Foundation of China under Grant LDT23F01015F01, in part by the Fundamental Research Funds for the Provincial Universities of Zhejiang Grant GK229909299001-008, in part by the Zhejiang Laboratory Open Research Project under Grant K2022QA0AB01.

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