Optimal production lot-sizing and condition-based maintenance policy considering imperfect manufacturing process and inspection errors

Abstract

In this paper, we propose an integrated Economic manufacturing quantity (EMQ) model combining both the concepts of condition-based maintenance (CBM) and imperfect manufacturing process. The manufacturing process is modelled by two indicators. One possesses binary state, indicating whether the manufacturing process is in-control or not. The other one is modelled by a homogeneous Gamma process, representing the degradation of the manufacturing equipment. The system is inspected at the end of each production run, upon which, the deterioration level can be perfectly observed, while two types of errors may occur in revealing the state of the manufacturing process. Defective products can be fabricated when the manufacturing state degrades. An integrated production and CBM policy is proposed. The objective is to develop the optimal production lot-sizing and preventive maintenance threshold in order to minimize the expected cost rate in the long-time horizon. We model the problem in the framework of a semi-Markov decision process. The successive-approximations method is applied to solve the problem numerically. The applicability of the proposed model and some sensitivity analysis are presented in a numerical illustration. It can provide theoretical reference to the decision-maker in production and maintenance planning.

Keywords: Economic manufacturing quantity; condition-based maintenance; imperfect manufacturing process; inspection error; semi-Markov decision process

1. Introduction

Planing production and maintenance are major issues that have been investigated extensively in manufacturing systems. In the classical economic manufacturing quantity (EMQ) models, the commonly considered problems are how to determine the lot sizing and inventory level, and how to schedule and sequence the production operations [1, 2, 3, 4]. In maintenance modelling, the objective is usually to minimize the maintenance cost or to maximize the system availability, where decisions are only based on the health condition of the system [5, 6]. In effect, production planning and maintenance are highly coupled [7, 8, 9, 10]. Production performance is usually interrupted by various uncertainties such as machine failures, material delays, human errors, quality failures, etc. Maintenance activities can restore the equipment from breakdown/degradation to the normal working state. It plays an important role in the smooth running of the production process. In addition, maintenance scheduling should be coordinated with production planning: when developing the maintenance policy, its interruption to the production process should be considered. It has been shown that the integrated consideration can lead to an increase of profit up to 40% theoretically [11] . It is beneficial to jointly consider the problem of production scheduling and maintenance planning.

In recent decades, condition-based maintenance has received enormous attention due to the development of the monitor technique and the costless sensor [12, 13, 14, 15]. It allows the decision-maker to schedule maintenance planning by utilizing data related to the system condition, which is more accessible comparing to failure data. Considering the joint optimization of the production and CBM scheduling problem, Peng and van Houtum [16] modelled the deterioration process of a manufacturing system by a continuous time and continuous state process, the average cost rate in the long-run was employed to assess the production and maintenance policy. Jafari and Makis [17, 18] studied a partially observable deteriorating system with discrete states, self-announcing failure and stochastic demand. They showed the advantages of conducting safety stock in minimizing the production and maintenance budget. Cheng et al. [19] focused on a multi-component system with structural and economic dependencies. A jointly Monte Carlo simulation technique and genetic algorithm were employed to search for the optimal lot size and preventive maintenance (PM) threshold. Based on the proportional hazards models with a continuous-state covariate process, Zheng et al. [3] considered the joint optimization of the lot size and the dynamic control limits in the framework of the semi-Markov decision process.

In most of the above studies, only one indicator is utilized to describe the health condition of the equipment. In effect, there exist many scenarios where utilizing one indicator is insufficient to describe the manufacturing process. On the one hand, a manufacturing system may consist of multiple components to complete its functionality synergistically, for instance, the redundancy system [20, 21, 22], the load-sharing system [23, 24, 25]. It is essential to consider the state of each component in the modelling of the degradation at the system level. On the other hand, the manufacturing process may experience various out-of-control scenarios due to various causes. Some may has multiple degradation states that can be observed by sensors. Some are less predictable with only failure state and working state. It is a general case where discrete and continuous quantities are simultaneously involved in assessing the system operating condition. For example, during the hardening process, the electric motor's bearing may fail due to the natural fatigue, component damage, over loading, or poor design [26]. Hence, to describe the condition of the manufacturing process, indicators in both discrete and continuous state spaces are employed in this study.

Imperfect inspection and production loss are considered in this work. Due to human errors, technology and cost limitations, etc., inspection errors are inevitable. It is more realistic to take them into account in production and maintenance control problems. In the literature, they are generally classified as the type-I error and type-II error [27, 28, 29, 30, 31]. The former indicates that false alarm signal shows up when the considered system is in its normal state. The latter implies that system failure is not revealed by the inspection such that we erroneously identify it as in its normal state. Both of them may cause negative effect on the production and maintenance management. Production loss exists extensively in the manufacturing process. Defective items can be fabricated when the machine state or the manufacturing process degrades. In this context, Cheng et al. [32] considered that the proportion of defective items was a function of the system deterioration which was modelled by a homogeneous Gamma process. In the work of Khatab et al. [33], they utilized two defective rates associating with the system degradation level to describe the quality loss. Rivera-Gomez et al. [34] considered that the defective rate was a function of the stage of the ageing process and the number of repairs.

In this study, we intend to develop an integrated EMQ and CBM model where two indicators are utilized to describe the manufacturing process. Inspection errors and production loss are also taken into account. With the above considerations, this model is more realistic and flexible in describing many manufacturing scenarios, where both discrete random variable and continuous-time degradation process are utilized. It is also more complex and challenging when decisions are made on the basis of two-dimensional information with errors. The quality loss depends on both the equipment degradation and the state of the manufacturing process. The main contributions of this study are as follows.

- We investigate the joint design of production and CBM control strategies.
- We utilize two indicators to describe the manufacturing process.
- The quality loss depends on both the equipment degradation and the state of the manufacturing process.
- The impact of imperfect inspections with two types of errors are taken into consideration in the joint design problem.

The rest of the paper is organized as follows. In Section 2, the model descriptions including the system characteristics, the manufacturing process, etc., are introduced. In Section 3, we present the CBM policy and formulate the assessment cost function in the semi-Markov decision process. The problem is solved numerically and an illustrative example is illustrated in Section 4. Finally, we make our conclusion in Section 5.

2. Model description

Consider a manufacturing machine that produces a single product with lot size Q in each production run. The production and demand rates are respectively p and d, $p > d$. The machine ceases to operate for $(p-d)Q/d$ time period whenever the lot sizing arrives at its target. Let τ be the production run, which means that $\tau = Q/d$. Once the inventory is empty, the manufacturing process starts over. To ensure the system functionality and control the production budget, the equipment is inspected at the end of each production run, where two types of information can be obtained.

One is related to the system degradation modelled by a homogeneous Gamma process. Let Y_t represents the deterioration of the system at time t with probability density function

$$\psi_{at,b}(x) = \frac{b^{at}}{\Gamma(at)} e^{-bx}, x \geq 0, \tag{1}$$

where Γ is a gamma function with

$$\Gamma(at) = \int_0^{\infty} v^{at-1} e^{-v} \mathrm{d}v.$$

Define σ_y as the first passage time corresponding to threshold y, $y \geq 0$, which is expressed as

$$\sigma_y = \inf\{t \geq 0, Y_t \geq y\}.$$

Let $G_{\sigma_y}(t)$ be the distribution function of σ_y, then it can be further expressed as follows:

$$G_{\sigma_y}(t) = \mathbb{P}(\sigma_y \leq t) = \mathbb{P}(Y_t \geq y) = \frac{\Gamma(at, by)}{\Gamma(at)}, \tag{2}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function with

$$\Gamma(a, x) = \int_x^{\infty} z^{a-1} e^{-z} \mathrm{d}z.$$

The corresponding density function of σ_y is defined as $g_{\sigma_y}(t)$,

$$g_{\sigma_y}(t) = \frac{a}{\Gamma(at)} \int_{by}^{\infty} (\ln(u) - \psi(at)) u^{at-1} \exp(-u) \mathrm{d}u, \tag{3}$$

with

$$\psi(u) = \frac{\mathrm{d}\ln(\Gamma(u))}{\mathrm{d}u}. \tag{4}$$

The other one is a random variable indicating whether the manufacturing process is identified as in-control or not. Define that

$$X_t = \begin{cases} 0, & \text{if the production process is identified as in-control at } t, \\ 1, & \text{if the production process is identified as out-of-control at } t. \end{cases}$$

Let T be the sojourn time in the in-control state. Its cumulative density function (CDF) and probability density function (PDF) are $F(\cdot)$ and $f(\cdot)$ respectively. Define α and β as the type I and type II errors respectively, which can be given as

$$\alpha = P(X_t = 1 \mid T > t). \tag{5}$$

$$\beta = P(X_t = 0 \mid T \leq t). \tag{6}$$

We will utilize the tuple $\left(X_t, Y_t\right)$ to represent the inspection information where X_t is a discrete random variable and $\{Y_t\}_{t\geq 0}$ is a continuous stochastic process. Hence, $\left(X_t, Y_t\right)$ contains the information about the health condition of the manufacturing process. The impact of the health condition of the equipment on the production is considered in this study. Suppose that whenever the degradation is severe, or the system is in the out-of-control state, defective items are fabricated. The defective rate r_d is given as

$$r_d = \begin{cases} r_{01}, & X_t = 0, Y_t > L, \\ r_{10}, & X_t = 1, Y_t \leq L, \\ r_{11}, & X_t = 1, Y_t > L. \end{cases} \tag{7}$$

Equation (7) indicates that all items are non-defective when the manufacturing process is in control and the degradation of the system is less than the threshold

L. Otherwise, defective items are inevitably fabricated with defective rates depending on (X_t, Y_t). It is reasonable to assume that $r_{01} < r_{11}$ and $r_{10} < r_{11}$.

For the above system, we have modelled the impact of the health condition of the equipment on the production. Due to ageing and degradation, defective items may occur and the idea production output is hard to achieve. We therefore propose a CBM policy for the joint production and maintenance optimization issue. We take the long-run expected cost occurred in the production and maintenance process as the objective function. The production lot-sizing Q and the condition-based maintenance threshold are employed as the decision parameters. Details are presented in the next.

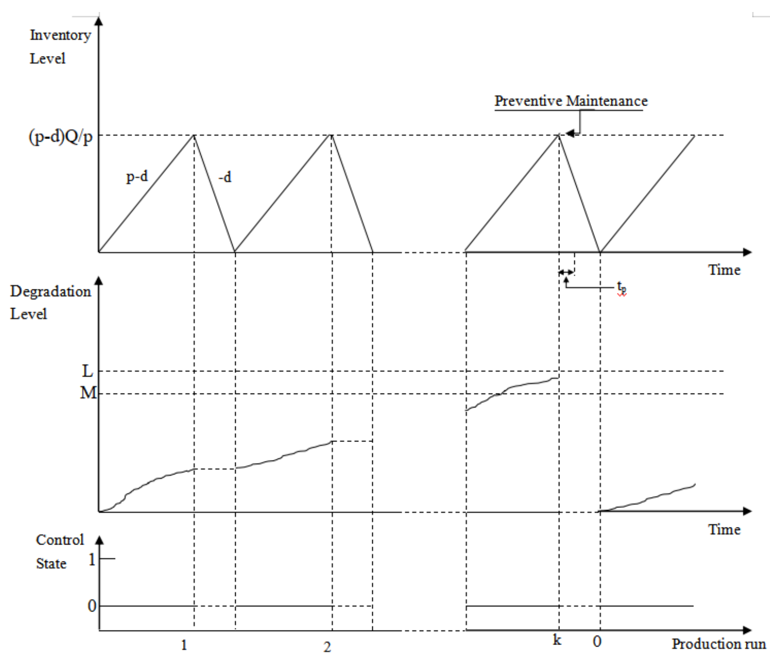

Figure 1: Illustration of preventive maintenance

3. Maintenance analysis

Assume that the production initiates from the perfect in-control state with zero machine deterioration. At the end of each production run, the degradation level is perfectly revealed and the control state of the production process is identified. Define the system state S_k as

$$S_k = \begin{cases} (k, y), & \text{if } X_{k\tau} = 0, Y_{k\tau} = y, \\ (\mathcal{F}, y), & \text{if } X_{k\tau} = 1, Y_{k\tau} = y, \end{cases} \tag{8}$$

where k signifies that the manufacturing process is regarded as in-control at the kth inspection. $\mathcal{F}$ implies that the manufacturing process is identified as out-of-control.

The maintenance policy is as follows.

case 1. If the degradation level y exceeds M, $0 < M < L$, the system is renewed immediately to state $(0, 0)$, meaning that the degradation of the system is restored to 0, and the production process is surely in-control as in the beginning of the manufacturing process.

case 2. If system in state $(\mathcal{F}, y), y \leq M$, the assignable cause is searched.

- If the out-of-control signal is validated, reactive maintenance is executed to restore the system to the in-control state $(0, y)$.
- If the alert is false, a compensatory maintenance is implemented to adjust the in-control process to a as-good-as-new state $(0, y)$.

case 3. Otherwise, no maintenance action is implemented and the machine will experience an idle time of period Q/d.

For simplicity, assume that both the control process and the degradation suspend during the idle period. Maintenance times are non-negligible. The production process starts over after the idle period or when maintenance is finalized, which occurs first. Figures 1 and 2 illustrate examples of case 1 and case 2 respectively. The system state they can be categorized as follows.

- state $(0, 0)$: the system is in its as-good-as-new state,
- state (k, y): the manufacturing process is identified as in-control at the kth inspection since its last renewaland the degradation level of the system is y, $k \neq 0, 0 < y \leq M$,

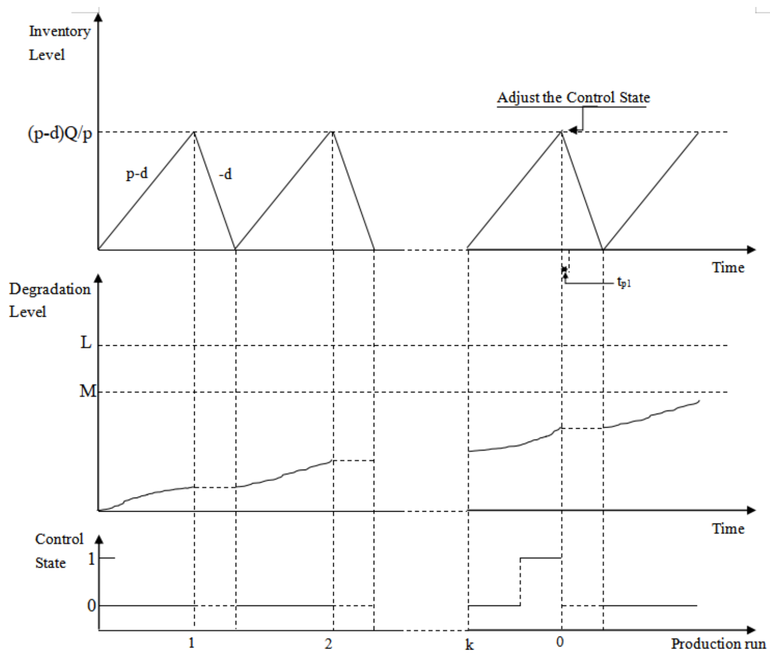

Figure 2: Illustration of adjustment of the control state

- state $(0, y)$: the system undergoes reactive or compensatory maintenance such that the control process is renewed and the degradation level of the system is $y, 0 < y \leq M$,
- state (PM_{10}, y): the control process is renewed by the compensatory maintenance due to the false alarm, $0 < y \leq M$,
- state (PM_{11}, y): the control process is renewed by the reactive maintenance due to the right detection, $0 < y \leq M$,
- state MS_1: the system is perfectly maintained when the degradation level is between M and L.
- state MS_2: the system is perfectly maintained when the degradation level exceeds L.

Therefore, the state space of the maintained systems is

$$\begin{aligned}\mathcal{S} &= \Big\{(0,0) \cup \{(k,y), k \in \mathbb{N}, 0 < y \leq M\} \cup \{(PM_{10}, y), 0 < y \leq M\} \\ &\quad \cup \{(PM_{11}, y), 0 < y \leq M\} \cup MS_1 \cup MS_2\Big\}.\end{aligned}$$

We consider the long-run expected cost occurred in the production and maintenance process as the objective function where the optimal lot-sizing and the PM threshold are determined. The problem is formulated in a semi-Markov decision process where costs occurred in the manufacturing process include the inspection cost, holding cost, shortage penalty and maintenance cost, etc. The cost structure is as follows.

- The inspection cost to reveal the system state: c_I
- The holding cost of each item is c_h
- The restoration cost of a defective item is c_d
- The time loss cost when the degradation level exceeds L is c_{l1} per unitary time.
- The time loss cost due to the maintenance is c_{l2} per unitary time.
- The compensatory maintenance cost due to the false identification of the system control state is c_{p0} with time t_{p0}
- The cost to restore the system from the out-of-control state to the in-control state is c_{p1} with time t_{p1}
- The preventive renewal cost of the system is c_p with time t_p
- The maintenance cost is c_f with constant time t_f when the degradation level exceeds L.

It is reasonable to assume that $c_f > c_p > c_{p1} > c_{p0}$ and $t_f > t_p > t_{p1} > t_{p0}$. In the next, given the system state space $\mathcal{S}$, the transition probabilities are calculated.

3.1. Transition probability

Due to the imperfect inspection with possible inspection error, define γ_k as the probability that the manufacturing process is in-control given that the system state is (k, y). Then γ_k can be represented as follows.

$$\gamma_k = \bar{\alpha}^k \bar{F}(k\tau), \tag{9}$$

where $\bar{\alpha} = 1 - \alpha$ with α the false alarm probability, $\bar{F} = 1 - F$ with F the cumulative density function of the sojourn time in the in-control process. γ_k equals to the probability that the process is always in control and no type I error has been made untill the kth inspection. Similarly, define η_k as the probability that the manufacturing process is out-of-control given that the system state is (k, y). It means that the manufacturing process shifts to the out-of-control state between $(i-1)\tau$ and $i\tau$ and it has not been revealed due to inspection errors until the kth inspection. Hence, η_k can be derived as follows.

$$\eta_k = \sum_{i=1}^{k} (F(i\tau) - F((i-1)\tau))\bar{\alpha}^{i-1}\beta^{k-(i-1)}, \tag{10}$$

where $\bar{\alpha} = 1 - \alpha$ with α the false alarm probability, β is the probability of type-II error.

Let $p_{A \to B}$ be the transition kernel corresponding to the state transition from A to B. First, consider that the initial state is (k, x), then the system may experience a production run without failure or alert, then

$$\begin{aligned} p_{(k,x)\to(k+1,y)} \quad = \quad & \Bigg[\frac{\gamma_k}{\gamma_k + \eta_k}\Big(\frac{F((k+1)\tau) - F(k\tau)}{\bar{F}(k\tau)}\beta + \frac{\bar{\alpha}\bar{F}((k+1)\tau)}{\bar{F}(k\tau)}\Big) \\ & + \frac{\beta\eta_k}{\gamma_k + \eta_k}\Bigg]\psi_{a\tau,b}(y - x). \end{aligned} \tag{11}$$

where γ_k and η_k have been presented in equations (9) and (10) respectively. $\psi_{a\tau,b}$ has been given in equation (4). $p_{(k,x)\to(k+1,y)}$ is the kernel that the degradation goes for x to y in time period τ, meanwhile, the manufacturing process is either in control and is well identified, or, it shifts to the out-of-control state at the $(k+1)$th inspection and type II error is conducted. Whenever a out-of-control signal is observed, the assignable cause is searched and if the alert turns out to be false, then

$$p_{(k,x)\to(PM_{10},y)} = \frac{\gamma_k}{\gamma_k+\eta_k}\frac{\alpha\bar{F}((k+1)\tau)}{\bar{F}(k\tau)}\psi_{a\tau,b}(y-x). \tag{12}$$

Otherwise,

$$p_{(k,x)\to(PM_{11},y)} = \left[\frac{\gamma_k}{\gamma_k+\eta_k}\frac{F((k+1)\tau)-F(k\tau)}{\bar{F}(k\tau)}\bar{\beta}+\frac{\bar{\beta}\eta_k}{\gamma_k+\eta_k}\right]\psi_{a\tau,b}(y-x). \tag{13}$$

Whenever the system is maintained, the transition probabilities can be further expressed as

$$p_{(k,x)\to MS_1} = G_{\sigma_{M-x}}(\tau)-G_{\sigma_{L-x}}(\tau), \tag{14}$$

$$p_{(k,x)\to MS_2} = G_{\sigma_{L-x}}(\tau). \tag{15}$$

where G is given in equation (2). Other transition kernels probabilities are

$$\begin{aligned} P_{(PM_{10},y)\to(0,y)} &= 1, \\ P_{(PM_{11},y)\to(0,y)} &= 1, \\ P_{MS_1\to(0,0)} &= 1, \\ P_{MS_2\to(0,0)} &= 1. \end{aligned} \tag{16}$$

In equations (11)-(16), ψ is the density function given in equation (1), γ_k and η_k have been presented in equations (9) and (10). G is given in equation (2). $\bar{\alpha} = 1-\alpha$ with α the false alarm probability, β is the probability of type-II error. $\bar{F} = 1-F$ with F the cumulative density function of the sojourn time in the in-control process.

3.2. Expected cost

First, the number of defected items during the manufacturing process with various initial system state is calculated. When the system stays in state (k, y), meaning that the manufacturing process is labelled as in-control at the kth inspection since its last renewal time and the degradation level is y. Let $N_d(k, y)$ be the expected number of the defected items until the next decision epoch given the initial system state (k, y), which can be further expressed by

$$N_d(k,y) = \frac{\gamma_k}{\gamma_k+\eta_k}N_d(k,y;\gamma_k)+\frac{\eta_k}{\gamma_k+\eta_k}N_d(k,y;\eta_k), \tag{17}$$

where $N_d(k, y; \gamma_k)$ represents the expected number of the defective items given that the manufacturing process is in the in-control process with all correct k inspections and the degradation level is y, $N_d(k, y; \eta_k)$ represents the scenario that the out-of-control is not identified due to the type II errors by the kth inspection. Given that the manufacturing process is in control with degradation level y, the number of expected defective items can be calculated as follows.

$$\begin{aligned} N_d(k, y; \gamma_k) &= r_{10} p \bar{G}_{\sigma_{L-y}}(\tau) \int_0^\tau \frac{f(v + k\tau)}{\bar{F}(k\tau)} (\tau - v) \mathrm{d}v \\ &+ r_{01} p \frac{\bar{F}((k+1)\tau)}{\bar{F}(k\tau)} \int_0^\tau (\tau - t) \mathrm{d}G_{\sigma_{L-y}}(t) \\ &+ r_{11} p \int_0^\tau \int_0^\tau \frac{f(v + k\tau)}{\bar{F}(k\tau)} (\tau - \max(v, t)) \mathrm{d}v \mathrm{d}G_{\sigma_{L-y}}(t) \end{aligned} \quad (18)$$

Equation (18) can be explained as follows. On the right-hand side, the first item represents the mean number of defective items when the process shifts to the out-of-control state and the degradation level is less thant L. The second item is the expected defective item when the manufacturing is in-control but the degradation level exceeds L. The third item is the expected defective items when the manufacturing process turns to be out-of-control and also, the degradation level exceeds L. For the scenario that the out-of-control state is not well identified due to type II error, $N_d(k, y; \eta_k)$ can be derived.

$$\begin{aligned} N_d(k, y; \eta_k) &= r_{10} p \tau \bar{G}_{\sigma_{L-y}}(\tau) + \int_0^\tau (r_{10} t + r_{11}(\tau - t)) \mathrm{d}G_{\sigma_{L-y}}(t) \\ &= r_{10} \int_0^\tau \bar{G}_{\sigma_{L-y}}(t) \mathrm{d}t + r_{11} \int_0^\tau G_{\sigma_{L-y}}(t) \mathrm{d}t \end{aligned} \quad (19)$$

In equations (18)-(19), r_{10}, r_{01}, r_{11} are the defective rates under different system states given in equation (7), $\bar{G} = 1 - G$ with G presented in equation (2), $\bar{F} = 1 - F$ with F the cumulative density function of the sojourn time in the in-control state, τ is the length of a production run, p is the production rate. By substituting equations (18)-(19) into equation (17), the expected number of defective items can be obtained.

Define $C_{\boldsymbol{s}}$ as the total expected cost during the manufacturing process from system state $\boldsymbol{s}$ until the next decision epoch, $s \in \mathcal{S}$. Then we have the following equations.

$$C_{(0,y)} = c_I + \frac{(p-d)\tau^2 c_h}{2} + c_d N_d(k,y) + c_{l1}\int_0^\tau G_{\sigma_{L-y}}(t)\mathrm{d}t \tag{20}$$

$$C_{(k,y)} = c_I + \frac{p(p-d)\tau^2 c_h}{2d} + c_d N_d(k,y) + c_{l1}\int_0^\tau G_{\sigma_{L-y}}(t)\mathrm{d}t, \quad k \neq 0. \tag{21}$$

The difference of equations (20) and (21) is that when $k > 0$, there exists extra inventory cost from the end of a production run to the run-out of the inventory. Whenever the maintenance actions are implemented, the corresponding expected total costs are presented as follows.

$$C_{(PM_{10},y)} = c_{p0} + c_{l2} d\max\big(t_{p0} - (\frac{Q}{d} - \tau), 0\big) + \frac{(p-d)^2\tau^2}{2d} c_h. \tag{22}$$

$$C_{(PM_{11},y)} = c_{p1} + c_{l2} d\max\big(t_{p1} - (\frac{Q}{d} - \tau), 0\big) + \frac{(p-d)^2\tau^2}{2d} c_h. \tag{23}$$

$$C_{MS_1} = c_p + c_{l2} d\max\big(t_p - (\frac{Q}{d} - \tau), 0\big) + \frac{(p-d)^2\tau^2}{2d} c_h. \tag{24}$$

$$C_{MS_2} = c_f + c_{l2} d\max\big(t_f - (\frac{Q}{d} - \tau), 0\big) + \frac{(p-d)^2\tau^2}{2d} c_h. \tag{25}$$

In equations (20)-(25), The expected cost includes the maintenance cost, the holding cost and the possible penalty when the maintenance time is long such that shortage occurs. r_{10}, r_{01}, r_{11} are the defective rates under different system states given in equation (7), $\bar{G} = 1 - G$ with G presented in equation (2), $\bar{F} = 1 - F$ with F the cumulative density function of the sojourn time in the in-control state, τ is the length of a production run, p and d are respectively the production rate and demand rate.

3.3. Expected sojourn time

Let $T_{\boldsymbol{s}}$ be the expected sojourn time until the next decision epoch when the initial system state is $\boldsymbol{s}, \boldsymbol{s} \in \mathcal{S}$. In effect, the expected sojourn time depends only on the degradation level of the equipment. Whenever the system state is $\boldsymbol{s} = (k, y)$, which means that the deterioration level is y at the end of the

kth production run since the as-good-as-new state of the state, for any $k \in \mathbb{N}$, $0 \leq y \leq M$, then the expected costs are

$$T_{(k,y)} = (\frac{Q}{d} - \frac{Q}{p})\boldsymbol{I}_{\{k \neq 0\}} + \frac{Q}{p}, \tag{26}$$

$$T_{(PM_{10},y)} = \max(\frac{Q}{d} - \frac{Q}{p}, t_{p0}), \tag{27}$$

$$T_{(PM_{11},y)} = \max(\frac{Q}{d} - \frac{Q}{p}, t_{p1}), \tag{28}$$

$$T_{MS_1} = \max(\frac{Q}{d} - \frac{Q}{p}, t_p), \tag{29}$$

$$T_{MS_2} = \max(\frac{Q}{d} - \frac{Q}{p}, t_f). \tag{30}$$

In equations (26) and (27), obviously that the expected time is equals to the run-out of the inventory from the beginning of a production run if $k = 0$. Otherwise, the expected time is the production run. In equations (28)-(30), the expected time is either $Q/d - Q/p$, or the end of the maintenance, which occurs first. $\boldsymbol{I}.$ is the indicator function, Q is the lot-size, p and d are respectively the production and demand rates, t_{p0}, t_{p1}, t_p and t_f are the maintenance cost presented in the cost structure. Hence, the expected sojourn time in each state can be obtained.

To calculate the long-run expected cost rate of the system, define $\pi_{\boldsymbol{s}}$ as the stationary distribution that the system stays in $\boldsymbol{s}, \boldsymbol{s} \in \mathcal{S}$, then the following balance equations can be obtained.

$$\begin{aligned}
\pi_{(k,y)} &= \pi_{(0,0)} p_{(0,0)\to(k,y)} \boldsymbol{I}_{\{k=1\}} + \int_0^y \pi_{(k-1,x)} p_{(k-1,x)\to(k,y)} \mathrm{d}x, \quad (31)\\
\pi_{(PM_{11},y)} &= \pi_{(0,0)} p_{(0,0)\to(PM_{11},y)} + \sum_{k\in\mathbb{N}} \int_0^y \pi_{(k,x)} p_{(k,x)\to(PM_{11},y)} \mathrm{d}x,\\
\pi_{(PM_{10},y)} &= \pi_{(0,0)} p_{(0,0)\to(PM_{10},y)} + \sum_{k\in\mathbb{N}} \int_0^y \pi_{(k,x)} p_{(k,x)\to(PM_{10},y)} \mathrm{d}x,\\
\pi_{(0,y)} &= \pi_{PM_{11},y} + \pi_{PM_{10},y},\\
\pi_{MS_1} &= \pi_{(0,0)} p_{(0,0)\to MS_1} + \sum_{k\in\mathbb{N}} \int_0^M \pi_{(k,y)} p_{(k,y)\to MS_1} \mathrm{d}y,\\
\pi_{MS_2} &= \pi_{(0,0)} p_{(0,0)\to MS_2} + \sum_{k\in\mathbb{N}} \int_0^M \pi_{(k,y)} p_{(k,y)\to MS_2} \mathrm{d}y.
\end{aligned}$$

with the normalization condition

$$\pi_{(0,0)} + \sum_{k\in\mathbb{N}} \int_0^M \pi_{(k,y)}\mathrm{d}y + \int_0^M \pi_{(PM_{11},y)}\mathrm{d}y + \int_0^M \pi_{(PM_{10},y)}\mathrm{d}y + \pi_{MS_1} + \pi_{MS_2} = 1. \quad (32)$$

Hence we can obtain $C_\infty(Q, M)$ as

$$C_\infty(Q,M) = \frac{\sum\limits_{\boldsymbol{s}\in\{(0,0),MS_1,MS_2\}} \pi_{\boldsymbol{s}} C_{\boldsymbol{s}} + \sum\limits_{k\in\{\mathbb{N},PM_{10},PM_{11}\}} \int_0^\tau \pi_{(k,y)} C_{(k,y)}\mathrm{d}y}{\sum\limits_{\boldsymbol{s}\in\{(0,0),MS_1,MS_2\}} \pi_{\boldsymbol{s}} T_{\boldsymbol{s}} + \sum\limits_{k\in\{\mathbb{N},PM_{10},PM_{11}\}} \int_0^\tau \pi_{(k,y)} T_{(k,y)}\mathrm{d}y}. \quad (33)$$

The optimal lot size and the preventive maintenance threshold can thus be obtained as

$$(Q^*, M^*) \quad = \quad \arg\min_{(Q,M)} C_\infty(Q, M).$$

We utilize the successive-approximations method in this study to solve equations (31) and to calculate the long-run expected cost rate with the help of equation (33). As the support of the deterioration state is continuous, we first discretize it into N states where N is a large integer and the discretize step is $\delta = \frac{M}{N}$. Hence the deterioration state is $\mathcal{S}^d = \{\delta, 2\delta, \cdots, M\}$ and the system is said to be in state k whenever its deterioration falls in $((k-1)\delta, k\delta]$. Besides, the sojourn time in the in-control state is finite with sojourn threshold T_{max} such that $1 - F(T_{max}) < \zeta$ where ζ is a predefined number that approximates 0. For a given production run τ, denoted by $n_{max}(\tau) = \lceil \frac{T_{max}}{\tau} \rceil$, where $\lceil x \rceil$ represents the ceil of x. Thus the state space of the system can be given by

$$\boldsymbol{S}_{total} = \mathcal{K} \times \mathcal{S}^d \cup (0,0) \cup MS_1 \cup MS_2 \cup PM_{10} \times \mathcal{S}^d \cup PM_{11} \times \mathcal{S}^d, \quad (34)$$

with $\mathcal{K} = \{0, 1, \cdots, n_{max}(\tau)\}$ and $\mathcal{S}^d = \{\delta, 2\delta, \cdots, M\}$. We can derive the stationary distribution and the function $C_\infty(Q, M)$ respectively. Algorithm 1 gives the detail in calculating $C_\infty(Q, M)$.

Algorithm 1: The calculation of $C_\infty(Q, M)$

Input : Parameters $a, b, \alpha, \beta, L, r_{01}, r_{10}, r_{11}$, p, d, τ, M c_h, c_d, c_I, c_{l1}, c_{l2}, c_{p0}, t_{p0}, c_{p1}, t_{p1}, c_p, t_p, c_f, t_f, ζ, ϵ

Initialization;

(1). Initialize $\pi_{\boldsymbol{s}}$ with constraint $\sum_{\boldsymbol{s}\in\boldsymbol{S}_{total}} \pi_{\boldsymbol{s}} = 1$;

(2). Determine $n_{max}(\tau)$ and $\boldsymbol{S}_{total}$ by equation (34);

(3). Update $\pi_{\boldsymbol{s}}$ with the discretized version of equation (31), noted as $\pi_{\boldsymbol{s}}^{new}$;

while $|\pi_{\boldsymbol{s}} - \pi_{\boldsymbol{s}}^{new}| > \epsilon$, *for some* $\boldsymbol{s} \in \boldsymbol{S}_{total}$, **do**

  Update $\pi_{\boldsymbol{s}}$: $\pi_{\boldsymbol{s}} \leftarrow \pi_{\boldsymbol{s}}^{new}$;

  Update $\pi_{\boldsymbol{s}}^{new}$: by the discretized version of equation (31);

end

Calculate $C_{\boldsymbol{s}}$ by equations (20)-(25),$\forall \boldsymbol{s} \in \boldsymbol{S}_{total}$;

Calculate $T_{\boldsymbol{s}}$ by equations (26)-(30),$\forall \boldsymbol{s} \in \boldsymbol{S}_{total}$;

Output: Calculate $C_\infty(Q, M)$ by equation (33).

4. Numerical illustrations

4.1. Model implication background

The proposed model can be considered in the context of a machining center [32], which is responsible for boring holes to match the housings of cylindrical gears. Suppose that the production rate $p = 20$ and the demand rate $d = 10$. The quality of the hole, measuring by its diameter and depth, is related to the degradation level of the boring cutter. It can be described by a homogeneous Gamma process with pdf given in equation (1). The shape parameter is $a = 1.5$ and the scale parameter is $b = 2$. Besides, the manufacturing process may become out-of-control. Assume that the sojourn time in the in-control state from its initial state follows a Weibull distribution with cumulative density function

$$F(t) = 1 - e^{-(\frac{t}{u})^v}. \tag{35}$$

Let $u = 5, v = 1.2$. It means that the process is more likely to enter into the out-of-control process due to ageing and usage. The expected sojourn time in the in-control is $u\Gamma(1+\frac{1}{v})$ which is 4.7 approximately. The probabilities of Type I and Type II errors are $\alpha = 0.05$, $\beta = 0.2$ respectively. Defective items are fabricated when the degradation level exceeds $L = 4$ or when the process is in the out-of-control state. The defective rates are $r_{01} = 0.05, r_{10} = 0.1, r_{11} = 0.2$ where $r.$ has been defined in equation (7). Other parameters of the cost units

Table 1: Parameters of cost units and maintenance times

Parameters	c_h	c_d	c_h	c_{l1}	c_{l2}	c_{p11}	t_{p11}
	0.2	2	10	20	5	30	0.5
Parameters	c_{p10}	t_{p10}	c_p	t_p	c_f	t_f	
	1.5	25	0.5	50	1	180	

and the maintenance time units are given in Table 1. The optimal production lot-sizing and the PM threshold are calculated with the help of Algorithm 1. The discretization step is 0.01 and we chose the precision $\epsilon = 10^{-4}$. The sojourn time of the manufacturing process in the in-control state is truncated with $\zeta = 10^{-4}$. The following results are obtained by utilizing Matlab2018b on a Windows 8 Core 64-Bits operating system.

4.2. Numerical analysis

4.2.1. Properties of the PM and system renewal probabilities

Table 2 demonstrates the preventive maintenance and renewal probabilities in the steady state with various preventive maintenance threshold given $Q = 50$. It is observed that both of the two probabilities show decreasing tendencies with M. The proportion of system renewal due to PM decreases with M too. Because with smaller M, the system inclines to enter into the preventive maintenance zone. As the equipment is always replaced when the degradation level exceeds L, one can imagine that the PM proportion decreases with M.

Table 2: Preventive maintenance and renewal probabilities with various M

M	1.5	1.6	1.8	2	2.3
PM probability	0.3010	0.2892	0.2665	0.2447	0.2129
Renewal probability	0.3173	0.3077	0.2902	0.2748	0.2545
M	2.6	2.8	3	3.2	3.5
PM probability	0.1806	0.1581	0.1344	0.1093	0.0691
Renewal probability	0.2372	0.2270	0.2178	0.2092	0.1976

4.2.2. Properties of the optimal lot-sizing and PM threshold

Figure 3 illustrates the corresponding expected cost in the long-run $C_\infty(Q, M)$. The minimal cost arrives at 19.2981 with optimum $M^* = 2.3$, indicating an equilibrium between over-preventive maintenance and under maintenance. Figure 4 shows the variations of the expected cost rate with respect to the preventive maintenance threshold M and the production run τ. The optimal lot size and the preventive maintenance threshold are thus $(Q^*, M^*) = (46, 2.3)$. In each production run, the expected degradation amount equals to 1.725. The equipment is highly likely to be replaced at the second production run. The lot-sizing is $Q^* = 46$, which will be run out at $Q^*/d = 4.6$, which can support the customer demand during maintenance without shortage.

4.2.3. Properties of the expected number of defective items

Figure 5 shows the expected number of defective items fabricated given initial system state (k, y) in the optimal scenario where $(Q^*, M^*) = (46, 2.3)$. It is observed that the mean number of defective items increases with both k and y. Because the sojourn time of the manufacturing process in the in-control state follows a Weibull distribution with $v > 1$. The equipment inclined to enter into the out-of-control state with ageing. Therefore, the number of defective items are expected to be an increasing function of k. Similarly, a larger degradation level y indicates a worse health condition of the system, which can contribute to the increase of defective products too.

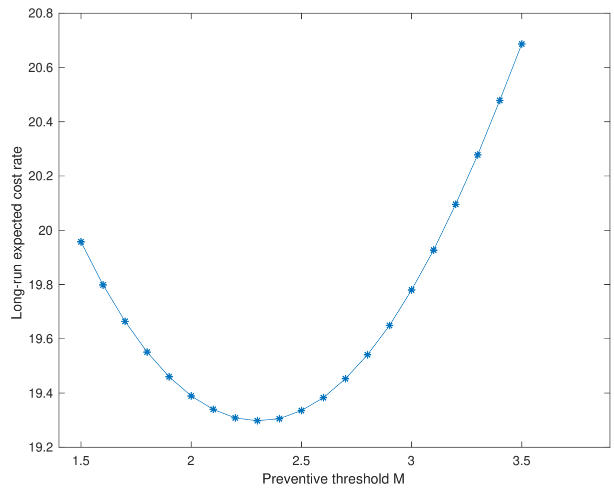

Figure 3: The variation of the long-run expected cost rate with different M.

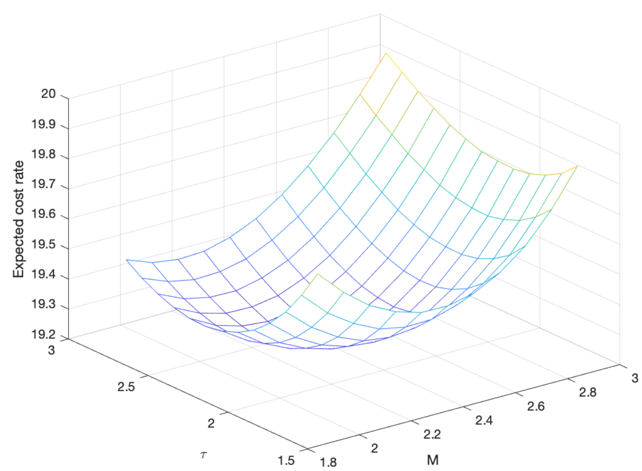

Figure 4: The variation of the long-run expected cost rate with different M and τ.

4.2.4. Sensitivity analysis

In the next, some sensitivity analysis are conducted to show the characteristics of the model. The variations of the optimal expected cost rate in the long-run and the corresponding production and maintenance policy are analyzed

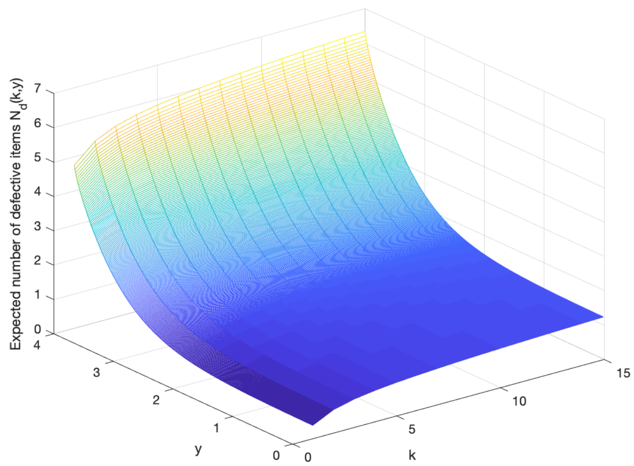

Figure 5: The expected number of defective items $N_d(k, y)$

in Table 3.

First, we consider the variation of L, which is the threshold that defective items are fabricated if the degradation level exceeds it. It is observed that for a smaller L, both the optimal lot size Q^* and the preventive threshold M^* are smaller. The decision-maker need to be more conservative in developing production and maintenance planning to mitigate the cost c_d due to defective products and cost c_{l1} due to the severe degradation of the equipment. Similarly, for a system with faster deterioration, the production run and the preventive maintenance threshold are smaller, where preventive maintenance is preferable with lower cost and the probabilities of higher defective cost and to be maintained with a large maintenance cost of the system become smaller.

For the indicator representing whether the manufacturing process is in control or not, a Weibull distribution with two parameters u and v is utilized to describe the sojourn time in the in-control state.Obviously that with a larger u, the manufacturing process is more stable and stays longer in the in-control state. Hence, less defective items are fabricated and the reactive/compensatory maintenance cost are expected to be lower. With a smaller $v, v > 1$, similar characteristics can be observed.

Table 3: The optimum cost rate and the corresponding policies under various critical parameters.

L	a	b	u	v	(Q^*, M^*)	$C_\infty(Q^*, M^*)$
4	1	2	5	1.2	(46, 2.3)	19.3886
3.5	1	2	5	1.2	(40, 2.2)	20.2790
4	1.5	2	5	1.2	(40, 2.1)	22.2990
4	1	2.2	5	1.2	(48, 2.4)	18.8377
4	1	2	10	1.2	(42, 2.4)	19.4184
4	1	2	5	1	(46, 2.3)	19.3604

5. Conclusions

In this study, we have designed an integrated EMQ and CBM control policy of a manufacturing system. We utilize two indicators to represent the health condition of the production process. One is modelled by a homogenous Gamma process, in continuous time with continuous state. The other one is described by a binary random variable. This is a general assumption where discrete and continuous quantities are simultaneously involved in assessing the system operating condition. We consider that the defective rate varies with the two indicators. Another novelty of this study is that inspection errors are considered. It is more realistic to consider the misidentification of the health condition as it seems inevitable in reality. For such a production system, we assess the production and maintenance policy by the long-run expected average cost rate, where the holding cost, cost due to defective production and maintenance cost are considered. The optimal lot-sizing and preventive maintenance threshold are decision parameters. The result may provide theoretical reference for the decision-maker when developing integrated production and maintenance strategy.

This model could be further extended in the following perspectives. First, we considered two indicators to represent the manufacturing system. It is more realistic and interesting to extend the study to the multi-indicator case, because a manufacturing system may fail to operate due to multiple competing failures. In addition, the assumption that indicators are independent is a strong condition. Stochastic dependence becomes a common phenomenon in production systems, especially the complex and Intelligent systems. Dependent indicators considering the coupling effect between the manufacturing process and the machine degradation is an interesting topic.

Secondly, the manufacturing equipment is assumed to be a single-component system. However, modern production systems are usually more and more complex and intelligent. When structure dependence is involved and all components contribute to the product quality, the optimal production and maintenance issue is more challenging. A lot of work can be extended in this regard.

Thirdly, imperfect inspection is considered where two types of inspection errors are assumed to be constant. In effect, the inspection errors may depend on the ageing or usage of the system. Time-dependent or usage-dependent inspection errors and the corresponding impact on the optimal integrated policy can be explored.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

Data will be made available on request.

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