Determining the optimal production-maintenance policy of a parallel production system with stochastically interacted yield and deterioration

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Abstract

In this paper, we study the integrated production-maintenance optimization problem of a multi-component deteriorating machine. The deterioration of each component is described by a discrete-time Markov chain with finite state space. To meet a constant demand, production planning is scheduled based on the system deterioration and the inventory level. We consider the mutual dependence between the production and the system deterioration. For each component, its deterioration transition probabilities depends on its own characteristics and its production quantity. Production yield is stochastically decreasing with the system deterioration. Maintenance is scheduled immediately if at least one component failure occurs. Otherwise, the decision-maker need to determine whether to schedule a preventive maintenance or to continue producing. We formulate the problem into a Markov decision process framework. The total discounted costs including the production costs, maintenance costs and holding/backlogging costs in the infinite horizon is obtained. Some structural properties of the optimal policy with respect to the machine condition, the inventory level are presented under mild conditions. The proposed model is further exam-

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ined by a numerical example, where the properties of the production and maintenance planning and the corresponding cost are investigated. It can provide theoretical reference in managing the production and maintenance problems in multiple production lines.

Keywords: Production-deterioration dependence; Random yield; Parallel production system; Joint optimization; Markov decision process

1. Introduction

In manufacturing engineering and management, lot-sizing is a critical quantity for manufacturers in developing efficient production strategies in order to maximize their profit or minimize the budget. For instance, the economic manufacturing quantity (EPQ) models [1, 2, 3, 4] have been extensively studied to achieve the above objectives. When considering the optimal production lot-sizing, random yield has received increasing attention in recent decades. Yield loss exists in various production in situ: in the Liquid Crystal Display (LCD) manufacturing process, yield loss can reach 50% or even higher [5]. Electronic fabrication/vibrations, environmental variations, equipment deteriorations, human errors in delivery, etc., can also result in yield with uncertainty. Scheduling production policies without taking the random yield into consideration may induce biased estimations of profit or unexpected holding/backlogging cost. Hening and Gerchak [6] presented a comprehensively analysis of a general periodical reviewed production/inventory model with random yield. Some structural properties of the cost and order point were examined with general production, holding/backlogging costs structures. Relevant works can be seen in [7, 8, 9, 10] where the impact of random yield on the production planning were examined. It is essential to take random yield into consideration in developing production schedules.

In the literature, one may notice that production scheduling and maintenance planning are usually studied separately. Production scheduling mainly focus on maximizing the profit or minimizing the budget by meeting the costumer demand and controlling the production lot-sizing, inventory cost, etc.[11, 12, 13, 14, 15]. A hidden assumption is that the manufacturing equipment never deteriorates/fails and can always fulfill its functionality. The study of machine maintenance always put emphasis on when to inspect, repair the system in order to minimize the relevant maintenance cost [16, 17, 18, 19] or to maximize the system availability [20, 21, 22, 23, 24, 25]. However, production and machine deterioration are usually mutual dependent. One can imagine that with the deterioration of the production equipment, the yield loss may become more and more significant. An example is the wear out of drill bits in a machine shop presented in [26]: due to the wear out of drill bits in a machine shop, product quality will deteriorate and the functioning components in a circuit manufacturing line may decrease. On the other hand, with a heavier production mission, the equipment may deteriorate faster, where many load-sharing systems can be regarded as examples. For instance, concerning the cascading failure in a power grid [27, 28], the transmission lines are more likely to be failed when its undertaking load is larger. Therefore, It is necessary to jointly consider the production and maintenance problems. In this framework, Sloan and Shanthikumar [29] investigated the integrated production and maintenance scheduling problem of a multiple-product, single-machine production system. Sloan [26] and Xiang et al. [30] examined the joint optimization of production quantity and maintenance planning, where the yield rate depended on the equipment condition. The former modeled the random yield with a binomial yield model and the latter assumed that the yield was stochastically proportional to the production input. Rather than focusing on establishing structural properties of the production and maintenance policy, Rokhforoz and Fink [31] considered a joint dynamic maintenance and production scheduling model, where they proposed an algorithm using model predictive control and Benders decomposition to solve the large-scale mixed-integer problem with coupling constraints. El Cadi et al. [32] investigated an integrated production and preventive maintenance (PM) problem with dependence between the system deterioration and production. They considered that the machine failure and product quality were operationdependent, where the age of the machine was a function of the number of items produced. Therefore, the increasing machine age leaded to an increasing system failure rate and an increasing proportion of defective products. uit het Broek et al. [33, 34] have considered the balance between the production rate and the system deterioration. It is seen that rarely work has been done with the consideration of the mutual dependence between the production and machine deterioration. In this study, we will take a step in this regard. The impact of the mutual dependence on the production and maintenance cost and the corresponding decision is examined.

Considering the configurations of the production systems, single-component systems are widely investigated, where a production mission is assigned to one single component. Considering multi-component production systems, traditional EPQ models with consideration of maintenance strategies have been proposed [35, 36, 37], where the optimal lot-sizing and preventive maintenance threshold were determined. However, the above studies assume that the lot-sizing is fixed and non-adjustable and the yield is perfect. Liu et al. [38] examined the condition-based maintenance for multi-component batch production where maintenance cost and system downtime were minimized. Their model focused on the maintenance problems, not on production scheduling.

In effect, as far as the authors know, no study has considered multi-component production systems with production-deterioration dependence and random yield. In this study, we intend to examine a parallel production system where components work together to satisfy the costumer demand. In jointly modelling of production and maintenance of parallel production systems, there exist several challenges. The first one is that when developing production plannings, the decision-maker needs to designate a specific workload to each component on the basis of the states of all components and the information of inventory level, customer demand, etc. It is more complex than dealing with the single-component system, where generally the size of the decision parameters is small. Secondly, stochastic dependence may exist among component failures and among production yields. Due to that components work together to satisfy the customer demand, one component may undertake more workload if some other components fail. In the work of Hao et al. [39], they discussed how to adjust the workload in a load-sharing system to mitigate the system failure and production loss. The yield rates of each component can be dependent due to the common working environment [16, 40]. In addition, maintenance of parallel production systems is complex: opportunistic maintenance can be implemented to components with severe degradations to minimize the interruption to the production process and to increase the system availability. On the other hand, during maintenance, it needs to determine the production activity of the non-maintained components. In this paper, we simplify the study by assuming that when maintenance is implemented, no production is allowed and the production is scheduled for all components immediately after the maintenance. This is possible in some circumstances, the whole system needs to be suspended when maintenance is implemented, even though some components can still operate physically. For instance, in continuous casting, a roller system consisting of multiple roller conveyors is subjected to heat, scaling, etc. It has high maintenance frequency and to implemented maintenance, it is necessary to stop the whole system to provide an accessible environment for the repairmen [41].

In this work, we consider the production and maintenance scheduling of a periodic-review, multi-component parallel production system. At each period, the policy decides whether or not to schedule maintenance (correctively or preventively) then to produce or how much to produce for each production unit if maintenance is not initiated. The problem is casted in to a Markov decision process framework [42, 43, 44] where we take the total discounted expected production and maintenance cost as the objective function. More details will be presented in the next section. The main contribution of this work are as follows.

- We study an integrated production-maintenance optimization problem of a parallel production system with random yield.
- We model and examine the dependence between production quantity and system deterioration.

- We investigate structural properties of the production and maintenance actions.
- The impact of random yield, production-deterioration dependence on the optimal production and maintenance planning are discussed.

The paper proceeds as follows. Section 2 introduces the problem descriptions, the system production, demand deterioration and the corresponding costs units, etc. In Section 3, we first consider a special case with two-component parallel system, where structural properties of the integrated policy is presented. Then a general N-component system is studied. Some numerical analysis are further given in Section 4 to demonstrate the characteristics and advantages of the proposed policy. We further present the conclusions and some future perspectives in Section 5.

2. Problem statement

We consider a N-component parallel production system that produces a single-product with interacted yield and system deterioration. The components can be heterogeneous with different production capacities and deteriorations. They work together to meet the product demand. Components are labelled as component 1, component 2, $\cdots$, component N, $S_o = \{1, 2, \cdots, N\}$. The deterioration of each component is modelled by a Markov chain with transition probability matrix depending on its own characteristic and the undertaken production load. The deterioration state set of component i is $S_i = \{1, 2, \cdots, M_i\}$, where "1" represents the perfect working state and M_i is the failure state of component i. Once component i enters into state M_i, it cannot leave this state unless maintenance intervention is executed. Let $X_n^{(i)}$ be the deterioration of component i in period n, $i \in S_o$, $n = 0, 1, 2, \cdots$. In this study, we suppose that the state transition probability of each component depends on its workload, i.e., its production input quantity. Given its production input quantity q, the transition probabilities are given as follows.

$$\mathbb{P}(X_{n+1}^{(i)} = l \mid X_n^{(i)} = v, q) = p_{vl}^{(i)}(q), \tag{1}$$

where $p_{vl}^{(i)}(q)$ satisfies

$$\sum_{l=j}^{M_i} p_{vl}^{(i)}(q) \leq \sum_{l=j}^{M_i} p_{ml}^{(i)}(q), \tag{2}$$

where $v \leq m$, $i \in S_o$, $q = 0, 1, \cdots$. Equation (1) implies that the deterioration of each component depends on its own characteristic and its input work load. Equation (2) indicates the increasing failure rate property of a component with multiple discrete states [45, 46]. It means that given the input quantity, each component inclines to go to worse condition if initially, its deterioration is more severe. In addition, $p_{vl}^{(i)}(q)$ is supposed to be non-decreasing with respect to q, meaning that with a heavier production load, the system condition deteriorates faster, $1 \leq v \leq l \leq M_i$.

A main novelty of this study is that we consider the interactions between production quantity and the deterioration process. Not only the production quantity has an impact on the system deterioration, the system deterioration also plays a role in the production realization. Given that the deterioration of component i is v and the input quantity is q, its output quantity $Y_q^{(i)}(v)$ is stochastically proportional to q. $Y_q^{(i)}(v)$ can be expressed as

$$Y_q^{(i)}(v) = U_v^{(i)} q, \tag{3}$$

where $U_v^{(i)}$ is a random variable on support $[0, 1]$. It is reasonable to assume that with a severe degradation state, the yield rate is lower stochastically, i. e., $\mathbb{E}U_v^{(i)} \geq \mathbb{E}U_l^{(i)}$ for $v \leq l, i \in S_o$, where $\mathbb{E}$ represents the expectation of the considered random variable. In addition, we call the yield is perfect if the output quantity equals to the input quantity, i.e. $Y_q^{(i)}(v) = q$.

Hence, we have modelled the interactions between the production and the deterioration of each component. The production capacity of component i is Q_i, for simplicity, assume that the capacities Q_i are all positive integers and the inputs are integers too. The product demand rate is assumed to be a constant d per period. Unsatisfied demands are backlogged.

At the beginning of each period, based on the states of the components and the inventory level, production and maintenance decisions are made. When maintenance is implemented, production is suspended for non-maintained components. It is then scheduled for all components immediately after the maintenance decision is made. If failure occurs at component level, failed component(s) should be maintained immediately. Meanwhile, it should be decided whether or not to perform preventive maintenance to the non-failed ones. If all components operate, the decision maker should decide whether to preventively maintain the deteriorated component(s), or to carry out production planning directly by determining the input production quantity for each production component. Maintenance is supposed to be perfect and instantaneous. The cost structure includes maintenance related cost, holding/backlogging cost and production cost that are given as follows.

The maintenance set-up cost is c_s. The preventive and corrective maintenance costs of component i are c_{pi} and c_{ci} respectively, $c_{pi} \leq c_{ci}, i \in S_o$. Given the inventory level s and the produce input quantity q, the production and holding/backlogging cost $h(s, q)$, where q is the total input production quantity which is the summation of the two production components. The objective is to determine the optimal production and maintenance schedule that minimizes the total expected discounted cost incurred during the production process.

3. Production and maintenance modelling and properties

In this section, we first investigate the explicit special case considering a two-component production system and then extend the results to general multiple component production systems.

3.1. Two-component production system

We utilize a triple (s, x_1, x_2) to represent the system state where s is the inventory level, x_1 and x_2 are the deterioration state of component 1 and 2 respectively. The state space is $\mathcal{R} \times S_1 \times S_2$. Given (s, x_1, x_2), the decision-maker can chose to perform maintenance then to produce or to produce and distribute a production quantity to each component. Therefore, the system state upon the subsequent period can be fully determined given the current system state and the production & maintenance action. We will derive the total expected discounted cost in the framework of the Markov decision process. Define $V(s, x_1, x_2)$ as the expected discounted production and maintenance cost in the infinite-time horizon. The Bellman equations can be expressed as

$$V(s,x_1,x_2) = \begin{cases} c_s + c_{c1} + c_{c2} + V(s,1,1), & \text{if } \quad x_1 = M_1, x_2 = M_2, \\ \min\{c_s + c_{c1} + V(s,1,x_2), c_s + c_{c1} + c_{p2} + V(s,1,1)\}, & \text{if } \quad x_1 = M_1, x_2 < M_2, \\ \min\{c_s + c_{c2} + V(s,x_1,1), c_s + c_{c2} + c_{p1} + V(s,1,1)\}, & \text{if } \quad x_1 < M_1, x_2 = M_2, \\ \min\big\{c_s + c_{p1} + V(s,1,x_2), c_s + c_{p2} + V(s,x_1,1), & \\ \quad c_s + c_{p1} + c_{p2} + V(s,1,1), \min\limits_{\substack{q_i \in [0,Q_i] \\ i=1,2}} J(s,x_1,x_2,q_1,q_2)\big\}, & \text{otherwise,} \end{cases} \tag{4}$$

where $J(s, x_1, x_2, q_1, q_2)$ satisfies

$$\begin{aligned} J(s,x_1,x_2,q_1,q_2) \quad = \quad & h(s-d, q_1+q_2) \\ & +\gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1 \hat{x}_1}(q_1) p^{(2)}_{x_2 \hat{x}_2}(q_2) \mathbb{E}_{1,2} V(s + U^{(1)}_{x_1} q_1 + U^{(2)}_{x_2} q_2 - d, \hat{x}_1, \hat{x}_2). \end{aligned} \tag{5}$$

γ is the discount factor. c_s is the set-up cost of maintenance, c_{pi} are c_{ci} are the preventive and corrective maintenance costs respectively. $h(\cdot, \cdot)$ is the holding cost.

Equations (4) and (5) can be explained as follows. Upon inspection, if at least one component enters into the failure state, then the failed one(s) should be correctively replaced immediately, meanwhile, the non-failed component can be either preventively replaced or left unchanged. The cost includes the maintenance set-up cost, replace cost. If the two components are operating, the decision-maker can chose to preventively replace the deteriorated component(s), or to produce directly, where the production input quantities for the two components need to be decided. $J(s, x_1, x_2, q_1, q_2)$ represents the expected production and maintenance cost of the next inspection when the system state is (s, x_1, x_2) and the production input of component i is q_i, $i = 1, 2$. $\mathbb{E}_{1,2}$ represents the expectation with respect to $U^{(1)}_{x_1}$ and $U^{(2)}_{x_2}$. $p^{(1)}_{x_1 \hat{x}_1}(q_1)$ and $p^{(2)}_{x_2 \hat{x}_2}(q_2)$ are the transition probabilities that have been given in equation (2). In the next, some characteristics of the optimal policy are presented.

Proposition 1. *$V(s,x_1,x_2)$ is non-decreasing in x_i, $i=1,2$.*

Proof. The value iteration is implemented to prove the monotonicity. We utilize a subscript to label each iteration. Suppose that $V_0(s,x_1,x_2)=0$ for all initial system state. Obviously that $V_0(s,x_1,x_2)$ is non-decreasing with x_1. Assume that at the kth interation, $V_k(s,x_1,x_2)$ is non-decreasing in x_1. Then at the $(k+1)$st iteration, $V_{k+1}(s,x_1,x_2)$ can be expressed as

$$V_{k+1}(s,x_1,x_2)=\begin{cases} c_s+c_{c1}+c_{c2}+V_{k+1}(s,1,1), & \text{if} \quad x_1=M_1, x_2=M_2,\\ \min\{c_s+c_{c1}+V_{k+1}(s,1,x_2), c_s+c_{c1}+c_{p2}+V_{k+1}(s,1,1)\}, & \text{if} \quad x_1=M_1, x_2<M_2,\\ \min\{c_s+c_{c2}+V_{k+1}(s,x_1,1), c_s+c_{c2}+c_{p1}+V_{k+1}(s,1,1)\}, & \text{if} \quad x_1<M_1, x_2=M_2,\\ \min\big\{c_s+c_{p1}+V_{k+1}(s,1,x_2), c_s+c_{p2}+V_{k+1}(s,x_1,1), & \\ \qquad c_s+c_{p1}+c_{p2}+V_{k+1}(s,1,1), \min\limits_{\substack{q_i\in[0,Q_i]\\ i=1,2}} J_{k+1}(s,x_1,x_2,q_1,q_2)\big\}, & \text{otherwise,} \end{cases} \tag{6}$$

where $J_{k+1}(s,x_1,x_2,q_1,q_2)$ satisfies

$$\begin{aligned} J_{k+1}(s,x_1,x_2,q_1,q_2) &= h(s-d,q_1+q_2) \\ &\quad +\gamma \sum_{\hat{x}_1\in S_1,\hat{x}_2\in S_2} p^{(1)}_{x_1\hat{x}_1}(q_1)p^{(2)}_{x_2\hat{x}_2}(q_2)\mathbb{E}_{1,2}V_k(s+U^{(1)}_{x_1}q_1+U^{(2)}_{x_2}q_2-d,\hat{x}_1,\hat{x}_2). \end{aligned} \tag{7}$$

We will prove that $J_{k+1}(s,x_1,x_2,q_1,q_2)$ is non-decreasing in x_1 for any $q_i\in\{0,1,\cdots,Q_i\}, i=1,2$. As $h(s,q)$ is independent of x_1, we only need to prove that the second item in equation (7) is non-decreasing in x_1. Due to that $p^{(1)}_{x_1\hat{x}_1}(q)$ has increasing failure rate, it is seen that $\sum\limits_{\hat{x}_1\in S_1,\hat{x}_2\in S_2} p^{(1)}_{x_1\hat{x}_1}(q_1)p^{(2)}_{x_2\hat{x}_2}(q_2)\mathbb{E}_{1,2}V_k(s+U^{(1)}_{x_1}q_1+U^{(2)}_{x_2}q_2-d,\hat{x}_1,\hat{x}_2)$ is increasing in x_1 with the assumption that V_k is increasing in x_1. Therefore, $J_{k+1}(s,x_1,x_2,q_1,q_2)$ is increasing in x_1. From equation (6), $V_{k+1}(s,1,x_2)$ and $V_{k+1}(s,1,1)$ are independent of x_1. It is only need to prove that $V_{k+1}(s,x_1,1)$ is non-decreasing in x_1. $V_{k+1}(s,x_1,1)$ can be expressed by

$$V_{k+1}(s,x_1,1)=\min\{c_s+c_{p1}+V_{k+1}(s,1,1), \min_{\substack{q_i\in[0,Q_i]\\ i=1,2}} J_{k+1}(s,x_1,1,q_1,q_2))\}.$$

Obviously that it is non-decreasing in x_1. Therefore, all items on the right-hand side of the last item in equation (6) are non-decreasing in x_1. We have shown that $V_{k+1}(s, x_1, x_2)$ is non-decreasing in x_1. Similarly, $V_{k+1}(s, x_1, x_2)$ is non-decreasing in x_2. Therefore, we can conclude that $V(s, x_1, x_2)$ is non-decreasing in x_i, $i = 1, 2$. $\diamond$

Based on the monotonicity of V, we can obtain the following proposition concerning the maintenance policy.

Proposition 2. *Given the system state (s, x_1, x_2), the maintenance policy has the following properties.*

- *If component 1 is preventively replaced, then it is also replaced for the system with initial state $(s, x^+, x_2), x^+ > x_1$.*
- *If component 2 is preventively replaced, then it is also replaced for the system with initial state $(s, x_1, x^+), x^+ > x_2$.*
- *If the two components are replaced, then they are also replaced for the system with initial state (s, x_1^+, x_2^+), $x_1^+ > x_1, x_2^+ > x_2$.*

The results can be derived directly with the help of Proposition 1 and it is thus omitted here. In the next, we present a property with respect to the inventory level. We assumed that unsatisfied demands are backlogged in this study and the inventory level s indicates the backlogged demand amount when $s < 0$. The following proposition shows the monotonicity of the value function when h is non-decreasing with s, meaning that the backlogging cost is non-decreasing with the backlogged amount.

Proposition 3. *When the yield is perfect, if $h(s, q)$ is non-increasing in s and non-decreasing in q when $s \leq 0$, then $V(s, x_1, x_2)$ is non-increasing in s for $s \leq 0$.*

Proof. When the output quantity is perfect, *i.e.* $U_x^{(i)} \equiv 1$, $i = 1, 2$. Let $L_k(s, x_1, x_2)$ represents the minimum of $J_k(s, x_1, x_2, q_1, q_2)$ with respect to q_1

and q_2. Then $L_{k+1}(s, x_1, x_2)$ can be expressed as

$$
\begin{aligned}
L_{k+1}(s, x_1, x_2) \quad = \quad & \min_{\substack{q_i \in [0, Q_i] \\ i=1,2}} h(s-d, q) \\
& + \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1 \hat{x}_1}(q_1) p^{(2)}_{x_2 \hat{x}_2}(q_2) V_k(s+q-d, \hat{x}_1, \hat{x}_2),
\end{aligned} \tag{8}
$$

where $q = q_1 + q_2$ representing the total yield. The value iteration is utilized to prove the theorem in the next. Let $V_0(s, x_1, x_2) = 0$ be the value function at beginning given any system initial state (s, x_1, x_2). Assume that V_k is decreasing when $s \leq 0$, we first prove that $L_{k+1}(s, x_1, x_2)$ is decreasing in s by the the following two scenarios.

(1). If $s \leq d - Q_1 - Q_2$, the value $s + q - d \leq 0$ for any q. So $V_k(s+q-d, x_1, x_2)$ is non-increasing with s. As $h(s, q)$ is non-increasing in s, form equation (8), we know that $L_{k+1}(s, x_1, x_2)$ is decreasing in s when $s \leq d - Q_1 - Q_2$.

(2). If $d - Q_1 - Q_2 < s \leq 0$, then there exist $\hat{q}$ such that $s + \hat{q} - d = 0$. For the input production less than $\hat{q}$, $L_{k+1}(s, \cdot, \cdot)$ is non-increasing in s. Hence, to find the minimum of $L_{k+1}(s, \cdot, \cdot)$, we can narrow the searching range to $[\hat{q}, Q_1 + Q_2]$. $L_{k+1}(s, x_1, x_2)$ can be written as

$$
\begin{aligned}
L_{k+1}(s, x_1, x_2) \quad = \quad & \min_{\substack{q_i \in [0, Q_i], i=1,2 \\ q_1+q_2 \in [\hat{q}, \hat{q}+1, \cdots, Q_1+Q_2]}} h(s-d, q_1+q_2) \\
& + \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1 \hat{x}_1}(q_1) p^{(2)}_{x_2 \hat{x}_2}(q_2) V_k(s+q_1+q_2-d, \hat{x}_1, \hat{x}_2),
\end{aligned} \tag{9}
$$

due to the assumption that V_k is non-increasing with s when $s \leq 0$. For system with initial state $(s-1, x_1, x_2)$, similarly, $L_{k+1}(s-1, x_1, x_2)$ can be expressed as follows.

$$
\begin{aligned}
L_{k+1}(s-1, x_1, x_2) \quad = \quad & \min_{\substack{q_i \in [0, Q_i], i=1,2 \\ q_1+q_2 \in [\hat{q}+1, \hat{q}+2, \cdots, Q_1+Q_2]}} h(s-1-d, q_1+q_2) \\
& + \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1 \hat{x}_1}(q_1) p^{(2)}_{x_2 \hat{x}_2}(q_2) V_k(s-1+q_1+q_2-d, \hat{x}_1, \hat{x}_2) \\
= \quad & \min_{\substack{q_i \in [0, Q_i], i=1,2 \\ q_1+q_2 \in [\hat{q}, \hat{q}+1, \cdots, Q_1+Q_2-1]}} h(s-1-d, q_1+q_2+1) \\
& + \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1 \hat{x}_1}(q_1) p^{(2)}_{x_2 \hat{x}_2}(q_2) V_k(s+q_1+q_2-d, \hat{x}_1, \hat{x}_2).
\end{aligned} \tag{10}
$$

Equations (9) and (10) are the minimum of J with q_1 and q_2 given in equation (8). The second equality in equation (10) holds when we replace $q_1 + q_2$ with $q_1 + q_2 + 1$. As h is non-increasing in s and non-decreasing in q, we know that $L_{k+1}(s, x_1, x_2) \leq L_{k+1}(s-1, x_1, x_2)$. From the statements in equations (1) and (2), we can conclude that $L_{k+1}(s, x_1, x_2)$ is decreasing in $s, s \leq 0$. Hence, $V_{k+1}(s, x_1, x_2)$ is decreasing in s and we can conclude that V is decreasing in s when $s \leq 0$. $\diamond$.

The following theorem can thus be obtained directly from proposition 3.

Theorem 3.1. *Assume that $h(s, q)$ is non-increasing in s and non-decreasing in q when $s \leq 0$. When the yield is perfect and the system is in the non-failed state, if the inventory level is negative, the optimal decision is either to perform maintenance then to produce or to schedule production directly with an amount that can restore the inventory to a non-negative level or to the maximum capacity if the inventory level is less than $d - Q_1 - Q_2$.*

In the next, we consider that $h(s, q)$ is expressed as

$$h(s, q) = c_0 q + c_h(s + q)\mathbb{I}_{\{s+q>0\}} - c_b(s + q)\mathbb{I}_{\{s+q<0\}} \tag{11}$$

where c_0, c_h and c_b are non-negative and finite numbers, $c_0 < c_b$. $\mathbb{I}$ is the indicator function. When h is defined as in equation (11), the following theorem can be derived.

Theorem 3.2. *If $V_0(\cdot) = 0$ for any system state, $\gamma < 1$. When the yields of component 1 and 2 are perfect, and the deteriorations are independent of the production amount, given the system state (s, x_1, x_2), the decision maker can either chose to schedule maintenance then to produce or to produce directly. If produce directly, the optimal total input quantity is $q^*(s, x_1, x_2)$, which is given as*

$$q^*(s, x_1, x_2) = \begin{cases} Q_1 + Q_2, & \textit{if } \frac{\partial K(q;s,x_1,x_2)}{\partial q} < -(c_0 + c_h), \\ 0, & \textit{if } \frac{\partial K(q;s,x_1,x_2)}{\partial q} > c_b - c_0, \\ max\big(0, min(Q_1 + Q_2, d - s)\big), & \textit{if } -(c_0 + c_h) \leq \frac{\partial K(q;s,x_1,x_2)}{\partial q} \leq (c_b - c_0), \end{cases}$$

where $K(q; s, x_1, x_2) = \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1\hat{x}_1} p^{(2)}_{x_2\hat{x}_2} \mathbb{E}V(s + q - d, \hat{x}_1, \hat{x}_2)$.

Proof. Let $K(q; s, x_1, x_2) = \gamma \sum_{\hat{x}_1 \in S_1, \hat{x}_2 \in S_2} p^{(1)}_{x_1\hat{x}_1} p^{(2)}_{x_2\hat{x}_2} \mathbb{E}V(s + q - d, \hat{x}_1, \hat{x}_2)$. It is seen that when the production input quantity and the deterioration of each component is independent, given the system state (s, x_1, x_2), $J(s, x_1, x_2, q_1, q_2)$ depends on q_1 and q_2 only through their summation $q_1 + q_2$, not on the specific value of each one. Thus we rewrite $J(s, x_1, x_2, q_1, q_2)$ as $J(s, x_1, x_2, q)$ in the next. The left derivative of $J(s, x_1, x_2, q)$ with q satisfies

$$\frac{\partial J(s, x_1, x_2, q)}{\partial q} = c_0 + c_h \mathbb{I}_{\{s+q-d>0\}} - c_b \mathbb{I}_{\{s+q-d<0\}} + \frac{\partial K(q; s, x_1, x_2)}{\partial q}, \quad (12)$$

Therefore the monotonicity of J can be divided into the following three scenarios.

(1). If $\frac{\partial K(q;s,x_1,x_2)}{\partial q} < -(c_0 + c_h), \forall q$, then we know that $J(s, x_1, x_2, q)$ is non-increasing in q and the minimum of $J(\cdot)$ achieves at $Q_1 + Q_2$.

(2). If $\frac{\partial K(q;s,x_1,x_2)}{\partial q} > c_b - c_h, \forall q$, then $\frac{\partial J(s,x_1,x_2,q)}{\partial q}$ is always non-negative and J is non-decreasing with q and thus $q^*(s, x_1, x_2) = 0$.

(3). If $-(c_0 + c_h) \le \frac{\partial K(q;s,x_1,x_2)}{\partial q} \le (c_b - c_0), \forall q$, then $d - s$ is the signal changing point of $J(\cdot)$ from negative to positive. Hence, if $d - s \in [0, Q_1 + Q_2]$, the optimum $q^*(s, x_1, x_2) = d - s$. Otherwise, $q^*(s, x_1, x_2) = 0$ if $d - s < 0$ and $q^*(s, x_1, x_2) = Q_1 + Q_2$ if $d - s > 0$. In a word, the optimum is $\max\big(0, \min(Q_1 + Q_2, d - s)\big)$. $\diamond$

3.2. General multi-component production systems

Consider a general N-component production system, let $(s, \boldsymbol{x})$ be the system state where s is the inventory level and $\boldsymbol{x} = (x_1, x_2, \cdots, x_N)$, $\boldsymbol{x} \in S_g = S_1 \times S_2 \times \cdots \times S_N$. x_i is the system deterioration with x_i representing the deterioration of component i, $i \in S_o$. Define $V(s, \boldsymbol{x})$ as the discounted optimal production and maintenance cost in the long-run. It satisfies:

$$V(s,\boldsymbol{x})=\begin{cases} c_s+\sum_{i=1}^{N}c_{ci}+V(s,\mathbf{1}), & \text{if} \quad x_i=M_i,\forall i\in S_o,\\ \min\{c_s+C(\boldsymbol{x},\boldsymbol{y})+V(s,\boldsymbol{y}),\forall \boldsymbol{y}\in A(\boldsymbol{x})\}, & \text{if} \quad S_c(\boldsymbol{x})\neq\emptyset,\\ \min\big\{\min\{c_s+C(\boldsymbol{x},\boldsymbol{y})+V(s,\boldsymbol{y}),\forall \boldsymbol{y}\in A(\boldsymbol{x})\},\ \min\limits_{\substack{q_i\in[0,Q_i]\\ i=1,2}} J(s,\boldsymbol{x},\boldsymbol{q})\big\}, & \text{otherwise,}\end{cases} \tag{13}$$

where $J(s,\boldsymbol{x},\boldsymbol{q})$ satisfies

$$J(s,\boldsymbol{x},\boldsymbol{q})=h(s-d,\sum_{i=1}^{N}q_i)+\gamma\sum_{\hat{\boldsymbol{x}}\in S_g}\Pi_{j=1}^{N}p^{(j)}_{x_j\hat{x}_j}(q_j)\mathbb{E}V(s+\sum_{v=1}^{N}U^{(v)}_{x_v}(q_v)-d,\hat{\boldsymbol{x}}), \tag{14}$$

with $\boldsymbol{q}=(q_1,q_2,\cdots,q_N)$, $\hat{\boldsymbol{x}}=(\hat{x}_1,\hat{x}_2,\cdots,\hat{x}_N)$. $\mathbf{1}$ is a $1\times N$ vector with 1s. c_s is the set-up cost of maintenance. $C(\boldsymbol{x},\boldsymbol{y})$ is the maintenance cost in the case that the system deterioration vector transfers from $\boldsymbol{x}$ to $\boldsymbol{y}$, expressed as:

$$C(\boldsymbol{x},\boldsymbol{y})=\sum_{i\in S_o}c_{pi}\mathbb{I}_{\{y_i=0\}}+\sum_{j\in S_c(\boldsymbol{x})}(c_{cj}-c_{pj})$$

with $\mathbb{I}$ as the indicator function. $A(\boldsymbol{x})$ consists of all possible system deterioration states after the corresponding maintenance action given the deterioration state $\boldsymbol{x}$. $h(\cdot,\cdot)$ is the holding cost. By equation (13), the discounted total production and maintenance cost can be obtained by the iteration algorithm numerically.

In effect, for the general case, the monotonicity of the value function in Proposition 1 still holds: it can be proved that $V(s,x)$ is non-decreasing with x_i, $i\in S_o$ if the increasing failure rate property in equation (2) holds for all components. A consequent property of the maintenance action is that, if component i is replaced with system state $(s,x_1,x_2,\cdots,x_i,\cdots,x_N)$, then it will also be replaced with system state $(s,x_1,x_2,\cdots,x_i^{+},\cdots,x_N)$ where $x_i^{+}>x_i$. Similarly, Proposition 3, Theorems 3.1 and 3.2 are properties of the value function with respect to the inventory level, they are also validated in the general multi-component production system.

4. Numerical illustration

4.1. Optimal production and maintenance policy of a two-component system

Consider a parallel production system with two deteriorating components. The state space of component i is $S_i = \{1, 2, 3, 4, 5\}$ with $M_i = 5$ representing the failure state of component i. The maximum production capacity of component i is $Q_i = 5$, $i = 1, 2$. The transition probabilities of component 1 are

$$p_{x\hat{x}}^{(1)}(q) = \begin{cases} 0.95 - 0.1 \times q, & \text{if } x = \hat{x},\, x < M_1, \\ 0.05 + 0.1 \times q & \text{if } \hat{x} = x + 1,\, x < M_1, \\ 0, & \text{otherwise,} \end{cases} \tag{15}$$

and $p_{M_1,M_1}^{(2)}(q) = 1$ for any $q \in \{0, 1, \cdots, Q_1\}$. For component 2, assume that

$$p_{y\hat{y}}^{(2)}(q) = \begin{cases} 0.9 - 0.1 \times q, & \text{if } y = \hat{y},\, y < M_2, \\ 0.1 + 0.1 \times q & \text{if } \hat{y} = y + 1,\, y < M_2, \\ 0, & \text{otherwise,} \end{cases} \tag{16}$$

and $p_{M_2,M_2}^{(2)}(q) = 1$ for any $q \in \{0, 1, \cdots, Q_2\}$. The deterioration of component 2 is faster than that of component 1 under each production input load q. Random yield depends on the deterioration of the system state and is supposed to be uniformly distributed, that is, $U_v^{(i)}$ is uniformly distributed on $[u_{min}^{(i)}(v), u_{max}^{(i)}(v)]$, $i = 1, 2$. Let $\boldsymbol{u}_{max}^{(i)} = [1, 0.9, 0.8, 0.7]$ be the vector where the vth element represents $u_{max}^{(i)}(v)$. Define $\boldsymbol{u}_{min}^{(i)} = [0.7, 0.6, 0.5, 0.2]$ where the vth element represents $u_{min}^{(i)}(v)$, $i = 1, 2$. It is seen that as the system deteriorates, the expected yield rate is expected to become lower. The demand per period is $d = 1$. Suppose that the production and holding/backlogging cost $h(s, q)$ is defined as in equation (11) where $c_0 = 4, c_h = 1, c_b = 10$. The maintenance cost units are $c_s = 1$, $c_{p1} = c_{p2} = 50$, $c_{c1} = c_{c2} = 200$. The discount factor is $\gamma = 0.9$. In this case study, the inventory space is truncated to $s \in [-40, 40]$ with discretized step $\delta_1 = 0.1$, where the inventory level are highly unlikely to reach out the above interval. We utilize the value iteration algorithm to find the value function in

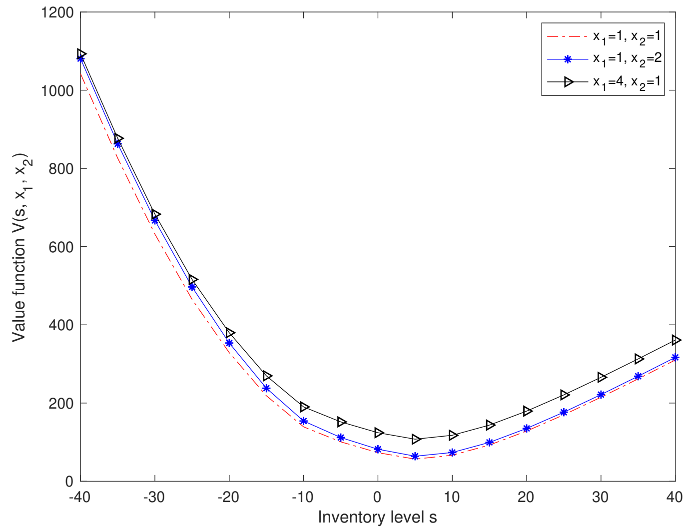


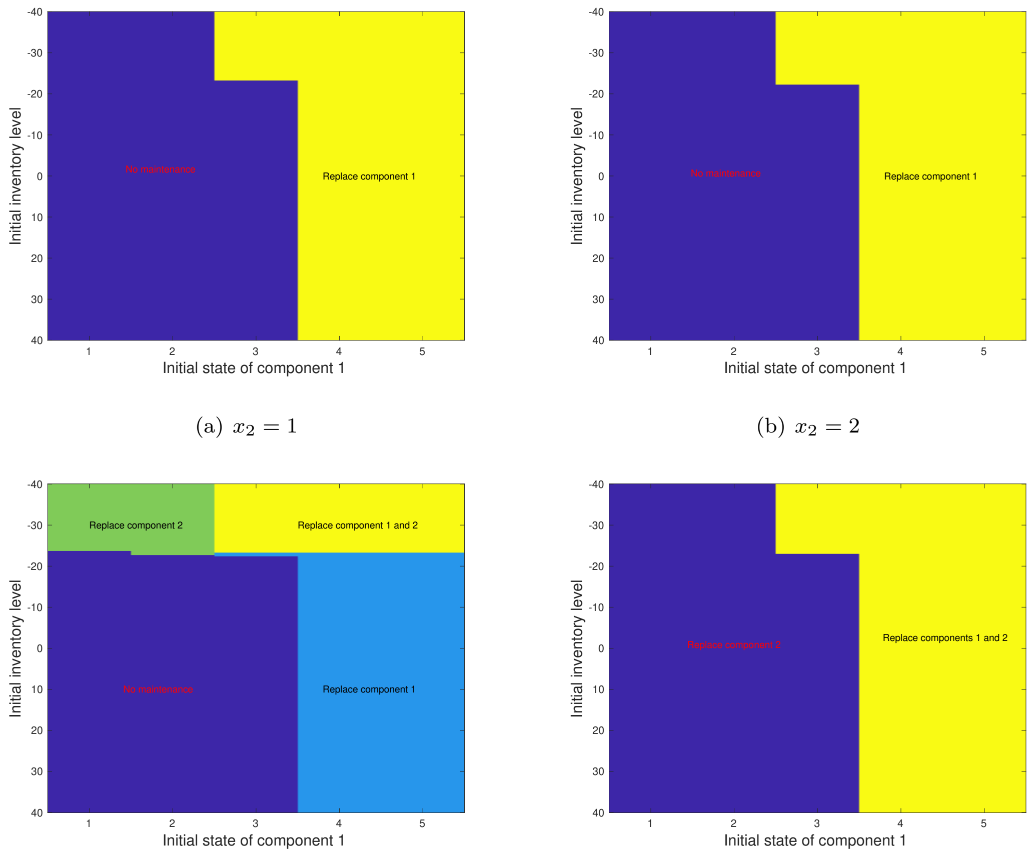
Figure 1: The variations of the value function with various initial system states

the next and the precision is $\epsilon = 0.001$. The following results are obtained by utilizing Matlab2018b on a Windows 8 Core 64-Bits operating system.

4.1.1. General results

Figure 1 shows the variations of the value function with various initial system states. As stated in Proposition 1, the value function is non-decreasing with the state of component i given the inventory level and the state of component $3-i$, $i = 1, 2$. It is also decreasing with s when $s < 0$, indicating the economic loss due to the backlogged production. In addition, when the deteriorations of the two components are settled, it seems that the value function is convex over the inventory level.

Figure 2 presents the characteristics of the optimal maintenance actions given initial system state (s, x_1, x_2). In each subfigure, x_2 is given and the maintenance actions changes with s and x_1. It can be observed as stated in Proposition 2 that, given (s, x_1, x_2), if component 1 is replaced, then it will also be replaced for system with initial state (s, x^+, x_2), $x_1 < x^+$. Similar property



(a) $x_2 = 1$

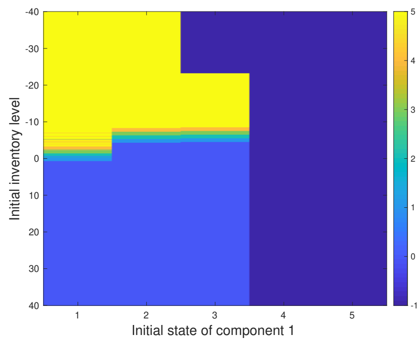
(b) $x_2 = 2$

(c) $x_2 = 3$

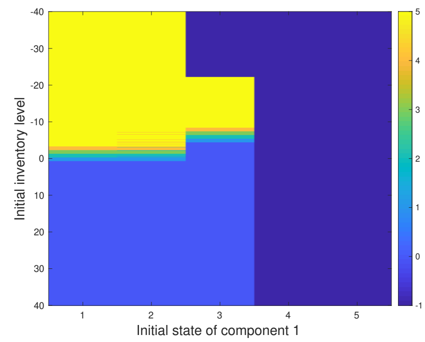
(d) $x_2 = 4$

Figure 2: The optimal maintenance actions with different initial inventory and deterioration levels

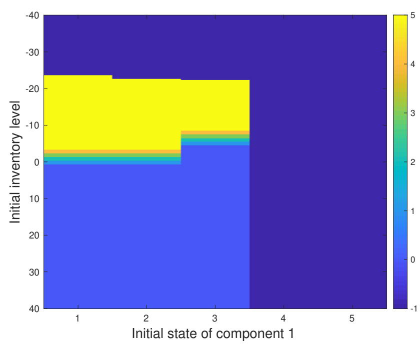
can be observed for component 2. The optimal action depends not only on the system deterioration, but also on the inventory level.



(a) $x_2 = 1$



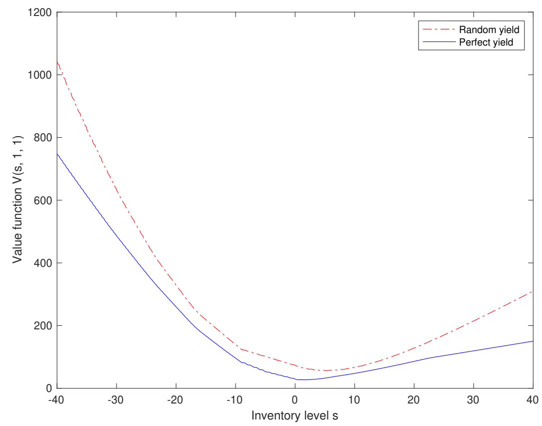
(b) $x_2 = 2$



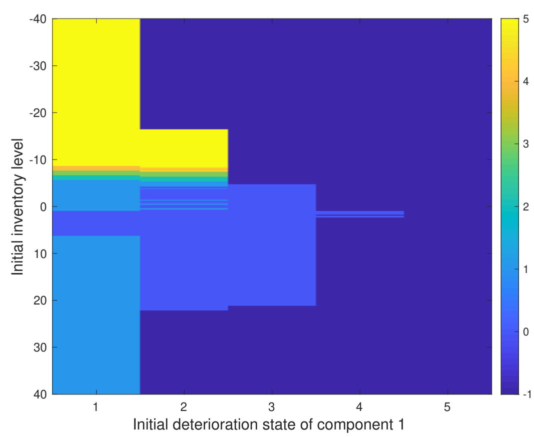
(c) $x_2 = 3$

Figure 3: The optimal production input quantity of component 1 under different initial inventory and deterioration levels

Figure 3 illustrates the production and maintenance planning of component 1 where “-1” indicates maintenance action is implemented. It is seen that in all the above cases, when the inventory level is negative, it is either to produce with the maximal capacity ($Q_1 = 5$) or to implement maintenance, which is coincide with Theorem 3.1.



(a) The variation of $V(s, 1, 1)$ with perfect yield and random yield respectively



(b) The production policy with various s and x_1 under perfect yield scenario

Figure 4: The value function $V(s, 1, 1)$ and the production policy with $(s, x_1, 1)$ under perfect yield scenario

4.1.2. Comparison results

Some comparison analysis is given in the subsequent paragraphs to illustrate the characteristics of the production-maintenance optimization problem.

(1). Comparison with the perfect yield case. Figure 4 presents the value functions $V(s, 1, 1)$ under random yield and perfect yield. It is seen that it is more cost-saving in the perfect yield scenario. The figure in case (b) gives the production policy of the perfect yield case when the initial state of component 2 is $x_2 = 1$. Comparing with the policy in the random yield situation, as illustrated in case (a) of Figure 3, maintenance is implemented earlier with smaller deterioration state of component 1 under perfect yield sceanario.

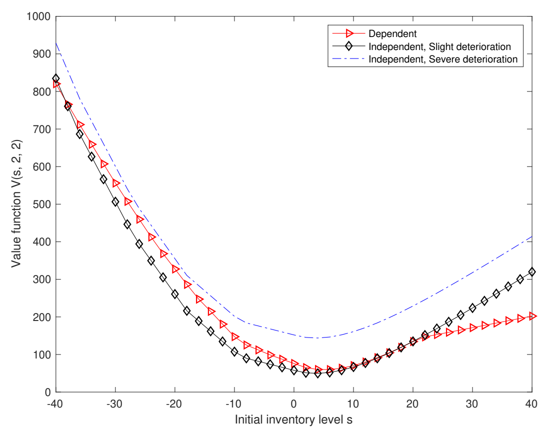
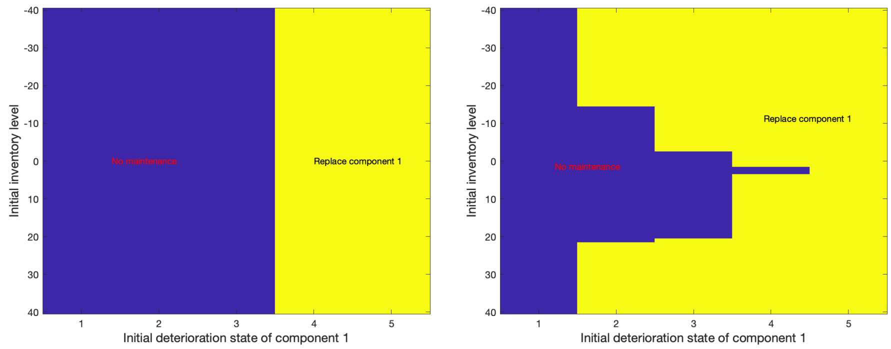


Figure 5: The variation of the value function with respect to s given that $x_1 = x_2 = 2$

(2). Comparison with the production-deterioration independent case. We consider the case when the production and system deterioration process are independent. Two independent cases are studied where the deterioration transition probability of component i is given as $\hat{p}^{(i)}_{x\hat{x}} = p^{(i)}_{x\hat{x}}(0)$ and $\hat{p}^{(i)}_{x\hat{x}} = p^{(i)}_{x\hat{x}}(3)$ where $p^{(i)}_{x\hat{x}}(q)$ has been presented in equations (15) and (16). We consider that the yield is always perfect in this comparison example. Figure 5 shows the variation of the value function $V(s, 2, 2)$ in different scenarios. It is seen that when the production and deterioration are independent, the value function is smaller when



(a) Maintenance policy of the PD-independent case

(b) Maintenance policy of the PD-dependent case

Figure 6: The maintenance policies of the PD-independent case (left. with slight deterioration) and PD-dependent case

the deterioration of the two components are slight, *i.e.* when $\hat{p}^{(i)}_{x\hat{x}} = p^{(i)}_{x\hat{x}}(0)$. In the dependent case, the probability transition depends on the production input quantity, which could be larger or smaller that the independent cases. The optimal maintenance policies under the two scenarios are given in Figure 6, where the deterioration state of component 2 is $x_2 = 2$. For the PD-independent case, maintenance policy is independent of the inventory level. For the PD-dependent case, it is seen that the maintenance depends on the system state and the inventory level.

4.1.3. Sensitivity analysis

In the next, some sensitivity analysis are conducted to show the variations of the production and maintenance cost and the corresponding policy with different parameters setting. We change the mentioned parameters each time and others are remain unchanged.

Figure 7 shows the variation of the value function with respect to the inventory level given $x_1 = 2, x_2 = 3$ under different system production capacities. Figure 8 presents the corresponding production input quantities for the two components where "-1" means that maintenance is implemented and no production is initiated. It is seen that when the inventory level is negative and

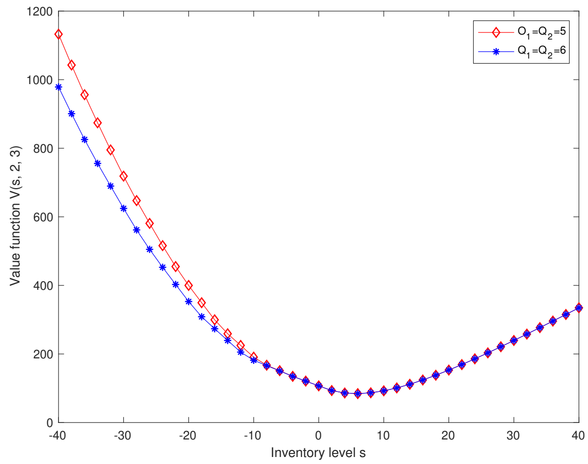


Figure 7: The variations of the value function $V(s, 2, 3)$ under different system production capacities

rather low, it is either to implement maintenance or to produce directly with input quantity that can restore the inventory level to the non-negative level. However, when the inventory is rather large, it seems that produce nothing ($q_1 = q_2 = 0$) and wait for the next decision epoch is the optimal action. In this situation, the capacity has no influence on the value function.

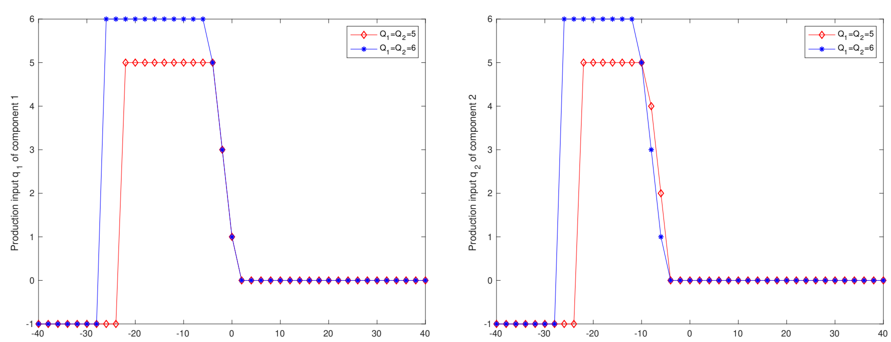


Figure 8: The variation of the production input quantity with respect to the inventory level given that $x_1 = 2, x_2 = 3$

Figure 9 presents the variation of the value function with respect to the inventory level and consumer demand. As expected, when the yield rate is higher, the cost induced in the production process is lower. When both the yield and demand are higher, the cost may become larger or smaller, depending on the initial inventory level.

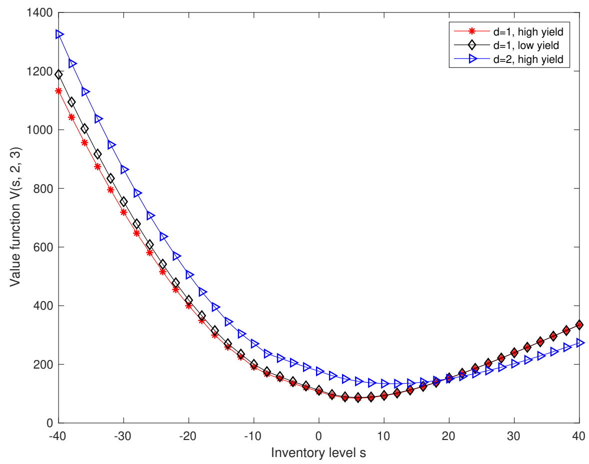


Figure 9: The variation of the value function with respect to the inventory level and demand given that $x_1 = 2, x_2 = 3$

4.2. Optimal production and maintenance policy of a three-component system

Table 1: The operating time with various production capacities

Production capacity	Operating time
$Q_i = 5,\ i = 1, 2, 3$	4462.1s
$Q_i = 4,\ i = 1, 2, 3$	2584.4s
$Q_i = 3,\ i = 1, 2, 3$	1370.6s

In the next, we consider a three-component production system. Assume that $M_i = 4$, $Q_i = 5$, $i = 1, 2, 3$. The transition probabilities of component 1 is given as in equation (15). Then transition probabilities of component 2 and component 3 are given as in equation (16). $U_v^{(i)}$ is uniformly distributed on

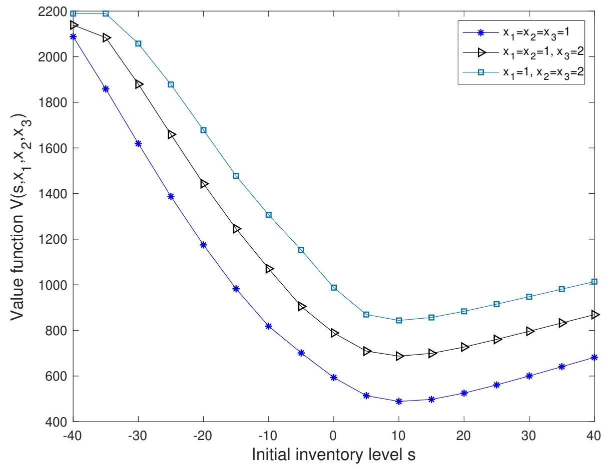


Figure 10: The variation of the value function with respect to the initial system state

$[u^{(i)}_{min}(v), u^{(i)}_{max}(v)]$, where $\boldsymbol{u}^{(i)}_{max} = [1, 0.9, 0.8]$ with the vth element representing $u^{(i)}_{max}(v)$. Similarly, $\boldsymbol{u}^{(i)}_{min} = [0.7, 0.6, 0.5]$ where the vth element represents $u^{(i)}_{min}(v)$, $v = 1, 2, 3$ and $i = 1, 2, 3$. The demand amount d is 1. The costs units, discount factor and discretized step are as given in the two-component system, which are $c_s = 1, c_0 = 4, c_h = 1, c_b = 10$, $c_{pi} = 50$, $c_{ci} = 200$, $i = 1, 2, 3$. The truncated inventory space is $[-40, 40]$, the discount factor is $\gamma = 0.9$, discretized step is $\delta_1 = 0.1$ and the precision is $\epsilon = 0.001$. Table 1 shows the operating time with various production capacities.

Figure 10 shows the variation of the value function $V(s, x_1, x_2, x_3)$ with the initial system state. Similar to the two-component system, the value function increases with the system deterioration. Table 2 further presents the value function and the corresponding production and maintenance policy. "DN" means no maintenance is implemented and "RE23" implies that components 2 and 3 are replaced. For each system state (s, x_1, x_2, x_3), concerning the maintenance policy, the control-limit property with the deterioration of each component can be observed. For the production scheduling, it depends on the inventory level and the system deterioration with no monotonic property. For instance, the production planning for system with deterioration $(1, 2, 3)$ can be produce with the full system capacity ($s = -35$, Q_i=5, i=1, 2, 3), or produce nothing and wait for the next period to decide ($s = 30$, $Q_i = 0, i = 1, 2, 3$) or to produce with partial capacity ($s = 0$, $Q_1 = Q_3 = 0, Q_2 = 1.$)

Table 2: Variation of the value function and the production and maintenance policy

Initial inventory	System deterioration	Value function	Maintenance & Production
-35	(1, 1, 1)	1858.7	DN, $Q_i = 5, i = 1, 2, 3.$
	(1, 2, 2)	2189.0	Re23, $Q_i = 5, i = 1, 2, 3.$
	(1, 2, 3)	2189.0	Re23, $Q_i = 5, i = 1, 2, 3.$
	(2, 3, 4)	2334.9	Re23, $Q_1 = 0, Q_2 = Q_3 = 5.$
-30	(1, 1, 1)	1619.2	DN, $Q_i = 5, i = 1, 2, 3.$
	(1, 2, 2)	2057.5	DN, $Q_i = 5, i = 1, 2, 3.$
	(1, 2, 3)	2189.0	Re23, $Q_i = 5, i = 1, 2, 3.$
	(2, 3, 4)	2334.9	Re23, $Q_1 = 0, Q_2 = Q_3 = 5.$
0	(1, 1, 1)	593.4	DN, $Q_1 = Q_3 = 0, Q_2 = 1.$
	(1, 2, 2)	987.7	DN, $Q_1 = Q_3 = 0, Q_2 = 1.$
	(1, 2, 3)	1341.2.0	DN, $Q_1 = Q_3 = 0, Q_2 = 1.$
	(2, 3, 4)	2334.9	Re23, $Q_1 = Q_3 = 0, Q_2 = 1.$
30	(1, 1, 1)	600.3	DN, $Q_i = 0, i = 1, 2, 3.$
	(1, 2, 2)	947.9	DN, $Q_i = 0, i = 0, 2, 3.$
	(1, 2, 3)	1343.9	DN, $Q_i = 0, i = 0, 2, 3.$
	(2, 3, 4)	2334.9	Re23, $Q_i = 0, i = 0, 2, 3.$

Table 3 presents the variation of the value function with the cost units where other parameters remain unchanged. As expected, the production and maintenance cost increases with the cost parameters.

5. Conclusion

In this paper, we have proposed an integrated production and maintenance optimization of a multi-component deteriorating production system For each

Table 3: Variation the value function with cost units

Costs units	System state	Value function
$c_{ci} = 150, i = 1, 2, 3$	(0, 1, 1, 1)	568.8
	(0, 1, 2, 2)	945.9
	(0, 2, 3, 4)	2232.4
$c_{pi} = 80, i = 1, 2, 3$	(0, 1, 1, 1)	606.9
	(0, 1, 2, 2)	1011.9
	(0, 2, 3, 4)	2402.5
$c_b = 8$	(0, 1, 1, 1)	520.0
	(0, 1, 2, 2)	862.8
	(0, 2, 3, 4)	2037.6
$c_0 = 6$	(0, 1, 1, 1)	650.2
	(0, 1, 2, 2)	1070.9
	(0, 2, 3, 4)	2511.8

component, its deterioration is described by a discrete-time Markov chain, depending on the production input quantity. The yield rate depends on the deterioration of the component, that is, the yield rate is stochastically decreasing with respect to the component deterioration level. At each period, based on the observed inventory level and the equipment condition, production and maintenance plannings are jointly determined. We have shown some structural properties of the value function and the optimal policy. The impact of the mutual dependence and other parameters on the cost function and the corresponding production and maintenance policy are further analyzed through a numerical example. The above consideration may provide some managerial insights to the decision-maker when developing production strategies. This work can be further extended in the following directions.

First, we have assumed that the deterioration of each component depends on its own characteristic and its production input quantity. It could be more realistic to consider the stochastic dependence among components due to the shared working condition. Secondly, we have considered a single product with one-stage manufacturing process with constant demand. The problem will become more complex if we consider multiple manufacturing process with random demand, where it takes multiple stages with stochastic component failures and random yields in each stage. Thirdly, the dependence between production and system health condition may be more complex and more studies in this directions are worth to be conducted. We have studied the scenario where the yield is stochastically proportional to the component deterioration. Some non-linear relations could be investigated.

In addition, maintenance time is supposed to be negligible in this work. In some production scenarios, for instance, for the offshore wind farm, maintenance delay is a practical issue and it is necessary to wait for a non-negligible time period before maintenance crew arrives. This is also an interesting topic.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability statement

Data will be made available on request.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (71901026).

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