Rule-based Composition Grammar Analysis and Applications

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Abstract

This paper proposes a visual method of melody writing based on music rules and the digital characteristics of Equal-temperament. Specifically, clock diagrams are used to visualize the equal ratio and cyclic relationship in Equal-temperament. After the visualization of all the keys used in the composition, this graphic method can be used to implement compose on any key. With the rule-based composition grammar, new melody can be written with chords as the basic unit through a visual "jump point" link. This grammar is only used for pitches in melody composition, and note duration will be studied in our future research. For those non-musicians, the rule-based composition grammar can be used to compose from simple to complex. The grammar analysis can be applied not only to melody, but also to chords, harmony, orchestration and so on.

1. Introduction

Composition is a difficult task for people without relevant experience on music [1]. To make the composition easier, the artificial intelligence (AI) techniques for music recognition, assisted composition and automatic composition, etc., have been increasingly applied in academic and professional field [2]. However, the composition process based on AI's bottom-up and emerging methods, often bring various problems on musical structure and basic composing rules [3]. From this point of view, a rule-based composition grammar is required and will be of great importance for a hybrid AI approach integrating machine learning and rule-based system together.

Although algorithms in current research of assistedcomposition can help people achieve so-called smart music composition. Especially for the people with little music education and training. Those algorithms pay to much attention on music decoration other than guiding people to understand the process of composition on musical syntax and structure. With such kind of AI tools, nonprofessional musician would grow dependency on algorithms and software, and hardly achieve independent musical creation with personal styles[4]. For example, AI algorithms could finish a whole piece of music based on several single notes, which contribute to creativity and the learning of composition.

Through abstracting the digital features of 12 Equal-temperament, this research focuses on establishing a framework for music digital representation and visualization, and making the music learning and practicing more rational and manipulatable. Another purpose of this research is to simplify the complicated music theory, and to provide a more practical and interesting way for composition beginners in music education [5].

2. Digitalization and Visualization for Chords

2.1. Equal-temperament Formalization

Equal-temperament is a temperament system that divides a group of octaves into twelve semitone intervals, such that the frequency ratio between the adjacent two is a constant [6]. The piano is a standard Equal-temperament instrument. The international standard pronunciation stipulates that the frequency of piano a1 is $440Hz$. According to the Equal-temperament, the frequency ratio of adjacent semitones is $2^{1/12} \approx 1.059463$. The order of a set of sounds is shown in Fig. 1. The formula for the frequency based on the Equal-temperament [7] is

$$f_n = 440 \times 2^{\frac{n-9}{12}}(Hz), n = 0, 1, \cdots, 11. \quad (1)$$

In this research, all pitch names (and the absolute frequency) are marked as: $0 - 1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - \alpha(10) - \beta(11)$. Since Equal-temperament

has been digitized, the Sharp Sign ♯ and the Flat Sign ♭ will not be distinguished in the following. Specifically, $\flat D/\sharp C, \sharp D/\flat E, \sharp F/\flat G, \sharp G/\flat A, \sharp A/\flat B$, will be uniformly written in the following format

$$C, \sharp C, D, \sharp D, E, F, \sharp F, G, \sharp G, A, \sharp A, B. \qquad (2)$$

Considering that the notes of Equal-temperament are cyclic

Pitch Name	C	#C bD	D	#D bE	E	F	#F bG	G	#G bA	A	#A	B
n	0	1	2	3	4	5	6	7	8	9	10	11
Frequency	f	$2^{\frac{1}{12}}f$	$2^{\frac{2}{12}}f$	$2^{\frac{3}{12}}f$	$2^{\frac{4}{12}}f$	$2^{\frac{5}{12}}f$	$2^{\frac{6}{12}}f$	$2^{\frac{7}{12}}f$	$2^{\frac{8}{12}}f$	$2^{\frac{9}{12}}f$	$2^{\frac{10}{12}}f$	$2^{\frac{11}{12}}f$
Approximation(Hz)	261.63	277.18	293.66	311.17	329.63	349.29	369.99	392.00	415.30	440.00	466.16	493.88

Figure 1. Correspondence among frequencies, pitch names and digits

[8], the notes arranged graphical are like a clock, as shown in Fig. 2. The twelve-division clock dials represent exactly the twelve-division relationship between the notes.

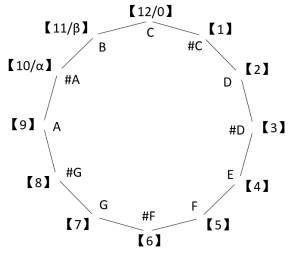


Figure 2. The twelve-division clock dial

2.2. Chords Visualization

Composition grammar involves lots of chords. Three most commonly used chords are us the Equal-temperament graphical model. For example, the chords I-V-IV in C Major on the keyboard is shown in Fig. 3. I-V-IV chord is connected to form a triangle over the diagram. The visual relationship between the three chords of I-IV-V can be found from Fig. 4.

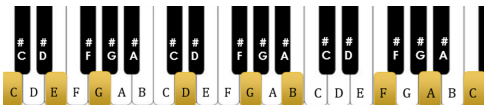
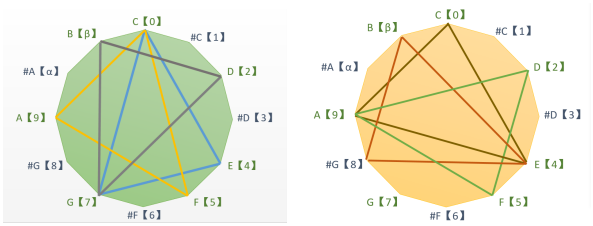


Figure 3. The chords I-V-IV in C Major on the keyboard

3. Basic Composition Grammar Based on Equal-temperament Diagram

Grammar of composition is complicated, which includes chords connection and melody composing [2]. This research aims at the melody composition which is based on a known chord progression. This is also the most significant part in composition for non-musicians. Based on published research [9], the basic composition grammar will be summarized in the following.



(a) I-V-IV of C Major (b) i-V-iv of A Minor

Figure 4. Triangles for chords in the clock dial

3.1. Chords Grammar

We identify each pitch in the chord via a 3-d vector (m, n, p) with associated frequencies satisfying $f_m < f_n < f_p$. Under this setting, the C Majorhords satisfy

$$\text{Major: } \begin{cases} \frac{f_n}{f_m} = 2^{\frac{4}{12}}; \\ \frac{f_p}{f_m} = 2^{\frac{7}{12}}. \end{cases} \qquad (3)$$

Correspondingly, the minor chords are:

$$\text{Minor: } \begin{cases} \frac{f_n}{f_m} = 2^{\frac{3}{12}}; \\ \frac{f_p}{f_m} = 2^{\frac{7}{12}}. \end{cases} \qquad (4)$$

Every major and minor chord in each key is listed in Fig. 5.

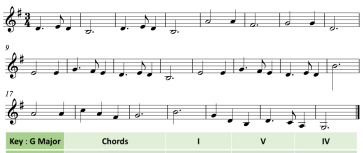
C Major	#C Major	D Major	#D Major	E Major	F Major	#F Major	G Major	#G Major	A Major	#A Major	B Major
I=047	I=158	I=269	I=37α	I=48β	I=590	I=6α1	I=7β2	I=803	I=914	I=α25	I=β36
V=7β2	V=803	V=914	V=α25	V=β36	V=047	V=158	V=269	V=37α	V=48β	V=590	V=6α1
IV=590	IV=6α1	IV=7β2	IV=803	IV=914	IV=α25	IV=β36	IV=047	IV=158	IV=269	IV=37α	IV=48β
C Minor	#C Minor	D Minor	#D Minor	E Minor	F Minor	#F Minor	G Minor	#G Minor	A Minor	#A Minor	B Minor
i=037	i=148	i=259	i=36α	i=47β	i=580	i=691	i=7α2	i=8β3	i=904	i=α15	i=β26
V=7β2	V=803	V=914	V=α25	V=β36	V=047	V=158	V=269	V=37α	V=48β	V=590	V=6α1
iv=580	iv=691	iv=7α2	iv=8β3	iv=904	iv=α15	iv=β26	iv=037	iv=148	iv=259	iv=36α	iv=47β

Figure 5. The chords I-V-IV in C Major on the keyboard

3.2. Phrase Grammar

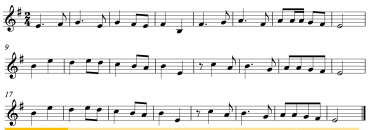
1. In a new generated phrase of music, both the start and end tone could be any of the three tones of a given chord. For example, the chord I (CEG) of C Major, that is: choose any note for beginning and ending as $a_j, a_j, j \in \{0, 4, 7\}$.

2. In addition to the first and last notes, the other notes follow the grammar as two situations. For example,



Key : G Major	Chords		I		V		IV
Beat: 3/4	Pitch Note (Diagram)		GBD (7β2)		D#FA (269)		CEG (047)
Melody	242β	242β	996	772	44764	242β	44764
Chord Connection	I	I	V	I	IV	I	IV
Melody	242β	99096	7β	72β	209	7	
Chord Connection	I	V	I	I	V	I	

(a) Silent night (Austrian Christmas Song)



Key : E Minor	Chords		i		V		iv
Beat: 2/4	Pitch Note (Diagram)		EGB (47β)		B#D#F (β36)		ADE (904)
Melody	4674	764	6β	6796	9976	4	B4
Chord Connection	i	i	V	V	V	i	i
Melody	(242)	0β9	β4	09	β7	9976	4
Chord Connection	(Ir)	iv	i	iv	i	V	i

(b) Katyusha (Russian folk song)

Figure 6. Examples for grammar analysis

the chord I (CEG) of C Major. 1) When reaching any note in the chord, that manifested as $a_j, j \in \{0, 4, 7\}$, the next note can be selected as $a_i, i \in \{0, 4, 7, a, b\}$

$$a = \begin{cases} j+1, j = 0, 1, 2, 3, 4, 5; \\ j-6, j = 6 \end{cases} \tag{5}$$

$$b = \begin{cases} j-1, j = 1, 2, 3, 4, 5, 6 \\ j+6, j = 0 \end{cases} \tag{6}$$

2) When reaching any note not in the chord, that manifested as $a_j, j \in \{1, 3, 5, 6\}$ the next note can be selected as $a_i, i \in \{j, a, b\}$,

$$a = \begin{cases} j+1, j = 0, 1, 2, 3, 4, 5 \\ j-6, j = 6 \end{cases} \tag{7}$$

$$b = \begin{cases} j-1, j = 1, 2, 3, 4, 5, 6 \\ j+6, j = 0 \end{cases} \tag{8}$$

3. The ending note of the last phrase of music need use the tonic tone of tonic chord (chord I / chord i) in the key. For example, 1) C Major: the whole song needs to end in tonic tone (C) in chord I; 2) A Minor: the whole song needs to end in tonic tone (A) in chord i.

This grammar can be verified in lots of music. Two of famous melody are selected to be the example, see Fig. 6.

4. Empirical Experiments

4.1. Graphic Coupling in Chord and Phrase Grammar

The above grammar could be easily applied into various scenarios with following steps.

1. As illustrated in Fig.7, when people start using this grammar, they firstly need to choose a music mode, e.g., the western seven-tone mode. Define a seven-note sequence that can be looped in the Equal-temperament diagram at first. Specifically, $a_j(j = 0, 1, 2, 3, 4, 5, 6)$ correspond to $a_0, a_1, a_2, a_3, a_4, a_5, a_6 = C, D, E, F, G, A, B$.

2. According to chords grammar, find and connect the three notes in chord I of C Major into a triangle.

3. According to phrase grammar 1 and 2, there are three methods of the new note selection: consecutive jumps, interlacing jumps and repeat, and two directions: clockwise and counterclockwise.

   1) Only when reaching the fixed point of the chord triangle, could consecutive jumps from one fixed point of the chord triangle to the other be applied. In this case, interlacing jumps and repeat can also be used. 2) If not reaching the fixed point of the chord triangle, interlacing jumps and repeat could be applied. 3) Clockwise and counter-clockwise may occur at any time.

4. According to phrase grammar 3, the note of the ending one in the music needs to chose C.

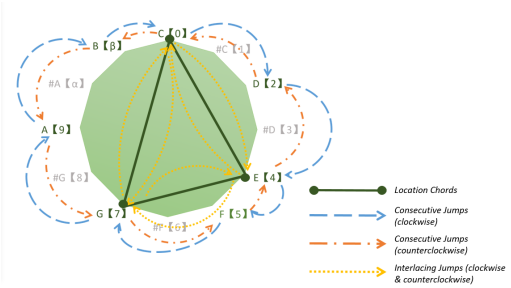


Figure 7. Illustration for the consecutive and interplacing jumps for note phrases

4.2. Composition Test

The composition grammar of this study is tested by more than 30 people without any music learning experience. With a given chords progression, they can write complete and beautiful melody through this grammar in a short time. Five melodies composed by this grammar for people with little

musical training are provided in Fig. 8. Subjects compose only the note pitches in the melody with the grammar of this research. The duration composition was done later by musicians. Moreover, the average score in the last row of each subject is evaluated by 3 musicians.

5. Conclusions

This research focuses on establishing a preliminary rule-based method on generating melodic pitches within a harmony system. It is very clear and simple to describe this grammar with an Equal-temperament diagram for non-musicians. Based on this composition grammar, In the future, it can also be expanded from single melody writing to harmony writing. The research can also support the development of graphical software for children and adults in music creation or learning. Usually, the aim of AI composition is to find the rules that are suitable for any situation. But the artistry and irregularity of music are unique to human beings. The method proposed in this research is to integrate both aspects, with the most creative characteristics of music, such as melodic tendency and identity, left for human.

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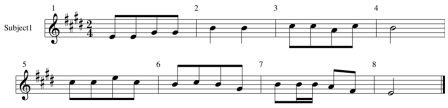
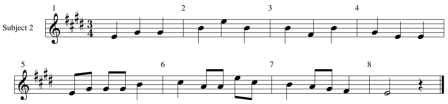
Key : E Major	Chords			I		V		IV
Beat:2/4	Pitch Note (Diagram)			E#GB (48β)		B#D#F (β36)		A#CE (914)
Chord Connection	I	I	IV	I	IV	I	V	I
Melody (subject1)	4488	ββ	1191	β	1141	β3β8	βββ96	4
Evaluation	The average score of musicians : 90.67 Well							



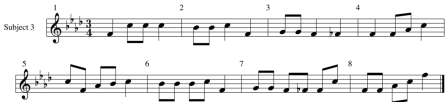
(a) Subject 1: Female,50 years old, retired teacher.

Key : A Major	Chords			I		V		IV
Beat:3/4	Pitch Note (Diagram)			A#CE (914)		E#GB (48β)		D#FA (269)
Chord Connection	I	I	V	I	I	IV	V	I
Melody (subject2)	911	494	4ββ	199	91114	62296	421β	9
Evaluation	The average score of musicians : 84.67 Good							



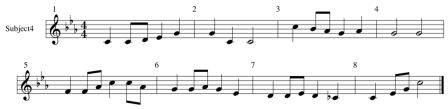
(b) Subject 2: Male,25 years old, staff of company .

Key : F Minor	Chords			i		V		iv
Beat:3/4	Pitch Note (Diagram)			F#GC(580)		CEG (147)		#A#CF (α15)
Chord Connection	i	iv	V	i	i	iv	V	i
Melody (subject3)	5000	αα05	7754	5580	058α0	αααα05	775441	55805
Evaluation	The average score of musicians : 94.33 Well							



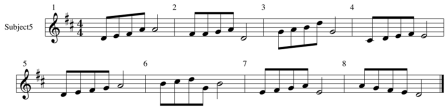
(c) Subject 3: Female, 32 years old, a child's mother .

Key : C Minor	Chords			i		V		iv
Beat:4/4	Pitch Note (Diagram)			C#DG (037)		GBD (7β2)		F#GC (580)
Chord Connection	i	i	iv	i	iv	i	V	i
Melody (subject4)	00237	700	0α878	77	558008	77873	2232β	0370
Evaluation	The average score of musicians : 89.66 Good							



(d) Subject 4: Male, 10 years old, student .

Key : D Major	Chords			I		V		IV
Beat:4/4	Pitch Note (Diagram)			D#FA (269)		A#CE (914)		GBD (7β2)
Chord Connection	I	I	IV	V	I	IV	V	I
Melody (subject5)	24699	66792	79β27	12464	24679	β127β	46794	97642
Evaluation	The average score of musicians : 81.33 Good							



(e) Subject 5: A group of 2 man and 2 women, 22-48 years old, diverse occupations.

Figure 8. Five melodies composed following the proposed grammar by 5 subjects.