学术报告

A curious identity on multiple sums over fields with applications

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Abstract： Let F be a field and let e and k be integers such that $1 \le e \le |F\backslash\{0\}|$ and $k \ge 0$. In this paper, we show that for any subset $\{a_1, \ldots, a_e\} \subseteq F\backslash\{0\}$, the following curious identity holds:

$$\sum_{\substack{(i_1,\ldots,i_e)\in Z_{\ge 0}^e \\ i_1+\ldots+i_e=k}} a_1^{i_1} \ldots a_e^{i_e} = \sum_{i=1}^{e} \frac{a_i^{k+e-1}}{\prod_{\substack{j=1 \\ j\ne i}}^{e}(a_i - a_j)}$$

with $Z_{\ge 0}$ being the set of nonnegative integers. As an application, we prove that for any subset $\{a_1, \ldots, a_e\} \subseteq F_q\backslash\{0\}$ with F_q being the finite field of q elements and *e* and *l* being integers such that $2 \le e \le q-1$ and $0 \le l \le e-2$, we have

$$\sum_{\substack{(i_1,\ldots,i_e)\in Z_{\ge 0}^e \\ i_1+\ldots+i_e=q-e+l}} a_1^{i_1} \ldots a_e^{i_e} = 0.$$

Then using this identity and providing an extension of the principle of cross-classification that slightly generalizes the one obtained by the speaker in 1996, we show that if *r* is an integer with $1 \le r \le q-2$, then$\{a_1, \ldots, a_r\} \subset F_q^*$, we have

$$\frac{x^{q-1}-1}{\prod_{i=1}^{r}(x-a_i)} = \sum_{i=0}^{q-1-r} \Big(\sum_{i_1+\ldots+i_r=q-1-r-i} a_1^{i_1} \ldots a_r^{i_r} \Big) x^i$$

This implies that

$$\#\left\{x \in \mathrm{F}_q^* \middle| \sum_{i=0}^{q-1-r} \left(\sum_{i_1+\cdots+i_r=q-1-r-i} a_1^{i_1} \dots a_r^{i_r} \right) x^i = 0 \right\} = q - 1 - r.$$

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