学术报告

Schrödinger equations with full or partial harmonic potentials, existence and stability results

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Venue： Room 111, Center for Applied Mathematics

Abstract：

This talk deals with ground states standing waves to the nonlinear Schrödinger equation with a partial confinement

$$\begin{cases} i\partial_t \phi + \Delta\phi - V(x)\phi = -\phi|\phi|^{p-1}, & (t,x) \in \mathbb{R}\times\mathbb{R}^N, \\ \phi(0,x) = \phi_0, \end{cases} \tag{0.1}$$

where $p \in (1, \frac{N+2}{N-2})$. By standing waves to (0.1) we intend solutions of the form $\phi(x,t) = e^{i\omega t}u(x)$ where thus u satisfies the stationnary equation

$$-\Delta u + V(x)u + \omega u = |u|^{p-1}u, \quad x \in \mathbb{R}^N. \tag{0.2}$$

A ground state is then a minimizer of the action functional associated to (0.2).

After a review of the classical case $V(x) \equiv 0$, we shall focus on situations where the potential V is either harmonic, $V(x) = x_1^2 + x_2^2 + \cdots + x_N^2$ (case of full confinement) or only partially harmonic, $V(x) = x_1^2 + x_2^2 + \cdots + x_{N-1}^2$ (case of partial confinement). In particular we shall prove that in some cases where the nonlinearity L^2-supercritical, namely $p \in (1 + \frac{4}{N}, \frac{N+2}{N-2})$, we obtain the existence of orbitally stable ground states. Our assumptions cover here the physically relevant cubic case and the equation we consider can be viewed as the limit case of the cigar-shaped model in Bose-Einstein condensate. We shall end with the presentation of some open problems.

This talk is based on joint works with J. Bellazzini, D. Bonheure, N. Boussaid, B. Noris and N. Visciglia.

欢迎大家参加！