A note on weighted Korn inequality

Manru Jiang & Renjin Jiang

November 27, 2017

Abstract: In this note, we show that, for domains satisfying the separation property, certain weighted Korn inequality is equivalent to the John condition. Our result generalizes previous result from [13] to weighted settings.

1 Introduction

Let Ω be a bounded, connected domain in $\mathbb{R}^n, n \geq 2$. Let $1 \leq p < \infty$, and $\rho(x)$ be the distance from x to the boundary $\partial\Omega$, i.e., $\rho(x) := \operatorname{dist}(x, \partial\Omega)$. For each $a \geq 0$, the space $L^p(\Omega, \rho^a)$ is the collection of functions f satisfying

$$\|f\|_{L^p(\Omega,\rho^a)} := \left(\int_\Omega |f(x)|^p \rho(x)^a\, dx\right)^{1/p} < \infty.$$

For every Sobolev vector filed $\mathbf{v} = (v_1, \cdots, v_n)$, let $D\mathbf{v}$ denotes its gradient matrix, $\epsilon(\mathbf{v})$ the linear part of the strain tensor, i.e.,

$$\epsilon_{i,j}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right),$$

and $\kappa(\mathbf{v}) = \{\kappa_{i,j}(\mathbf{v})\}_{1\leq i,j\leq n}$ be the anti-symmetric part of $D\mathbf{v}$ as $\frac{1}{2}(D\mathbf{v} - D\mathbf{v}^T)$.

Due to its fundamental role in the development of linear elasticity, Korn's inequality has been investigated intensively, see [1, 2, 6, 8, 9, 10, 12, 16, 19] and the references therein. The Korn inequality has been extended to John domains (see Section 2 for the definition) by Acosta, Durán and Muschietti [2], and was proved by Jiang-Kauranen [13] that the Korn inequality also implies the John condition if the domain additionally satisfies a so-called separation property (see Section 2), which was introduced by Buckley-Koskela [4]. The Korn inequality with weights on John domains were studied in [7, 14, 17]. In this note, we show that, under the separation property, certain weighted Korn inequality also implies the John condition.

[0] *Key words and phrases.* Korn inequality, John domain, weight, separation property

Theorem 1.1. *Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 2$. If Ω satisfies the separation property, then the following conditions are equivalent:*

(i) Ω is a John domain;

(ii) for some $p \in (1, \infty)$ and some $a \in [0, \infty)$, for all $\mathbf{u} = (u_1, \cdots, u_n) \in W^{1,p}(\Omega, \rho^a)$ satisfying $\int_\Omega (\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})\rho^a \, dx = 0, 1 \leq i, j \leq n$, it holds that

$$\|D\mathbf{u}\|_{L^p(\Omega,\rho^a)} \leq C_K \|\epsilon(\mathbf{u})\|_{L^p(\Omega,\rho^a)}; \tag{$K_{p,a}$}$$

(iii) $(K_{p,a})$ holds for all $p \in (1, \infty)$ and $a \in [0, \infty)$.

Notice that, on the plane, any finite connected domain satisfies the separation property, see [13, Section 6] for a detailed discussion of the separation property.

The note is organized as follows. In Section 2, we first recall the definitions of Sobolev spaces and John domain, then we recall some known result on weighted Korn inequalities. In Section 3, we provide the proof for Theorem 1.1. Throughout the paper, we denote by C positive constants which are independent of the main parameters, but which may vary from line to line. For matrices $S \in \mathbb{R}^{n\times n}$ we use the norm $\|S\| := \max\{|s_{i,j}| : 1 \leq i, j \leq n\}$.

2 Preliminaries

2.1 Sobolev spaces

Let $\mathscr{D}(\Omega)$ denote the set of smooth functions compactly supported in Ω. Let $u \in L^1_{\mathrm{loc}}(\Omega)$ and $1 \leq i \leq n$, $f_i \in L^1_{\mathrm{loc}}(\Omega)$ is a weak partial derivative of u, i.e.,

$$\int_\Omega u(x) \frac{\partial}{\partial x_i} \phi(x) \, dx = -\int_\Omega f_i(x)\phi(x) \, dx$$

holds for each $\phi \in \mathscr{D}(\Omega)$. In what follows, we will denote such f_i by $\frac{\partial}{\partial x_i} u$. For $p \in [1, \infty)$ and $a \geq 0$, the weighted Sobolev space $W^{1,p}(\Omega, \rho^a)$ is then defined as the set of all $u \in L^p(\Omega, \rho^a)$ with $\nabla u \in L^p(\Omega, \rho^a)$. For $u \in W^{1,p}(\Omega, \rho^a)$, define its norm by

$$\|u\|_{W^{1,p}(\Omega,\rho^a)} := \|u\|_{L^p(\Omega,\rho^a)} + \|\nabla u\|_{L^p(\Omega,\rho^a)}.$$

The Sobolev space $W^{1,p}_0(\Omega, \rho^a)$ is then defined as the completion of $\mathscr{D}(\Omega)$ with respect to the norm of $W^{1,p}(\Omega, \rho^a)$. We denote $W^{1,p}(\Omega, \rho^a)$ (resp. $W^{1,p}_0(\Omega, \rho^a)$) by $W^{1,p}(\Omega)$ (resp. $W^{1,p}_0(\Omega)$) if $a = 0$. Notice that as ρ^a is continuous positive functions in Ω, the subspace $C^\infty(\Omega) \cap W^{1,p}(\Omega, \rho^a)$ is dense in $W^{1,p}(\Omega, \rho^a)$; see [11, Theorem 3].

Let $p \geq 1$ and $b \geq a \geq 0$. We say that the $(P_{p,a,b})$-Poincaré inequality holds, if there exists $C > 0$ such that for every $u \in W^{1,p}(\Omega, \rho^a)$, it holds

$$\int_\Omega |u(x) - u_{\Omega,a}|^p \rho(x)^a \, dx \leq C \int_\Omega |\nabla u(x)|^p \rho(x)^b \, dx, \tag{$P_{p,a,b}$}$$

where we denote by $u_{\Omega,a} := \frac{1}{\int_\Omega \rho^a \, dx} \int_\Omega u\rho^a \, dx$ and $u_\Omega := u_{\Omega,a}$ for $a = 0$.

2.2 John domain and weighted Korn inequality

In this subsection, let us recall the definition of John domain and some known results on weigthed Korn inequalities. This class was introduced by F. John in [15], then named after John by Martio and Sarvas in [18].

Definition 2.1 (John domain)**.** A bounded domain $\Omega \subset \mathbb{R}^n$ with a distinguished point $x_0 \in \Omega$ is called a John domain if it satisfies the following "twisted cone" condition: there exists a constant $C_J > 0$ such that for all $x \in \Omega$, there is a curve $\gamma : [0, \ell] \to \Omega$ parametrised by arclength such that $\gamma(0) = x, \gamma(\ell) = x_0$, and $d(\gamma(t), \Omega^c) \geq C_J t$ for each $t \in [0, \ell]$. We call such a curve a John curve for x.

Notice that, any point can act as the distinguished point in the John domain, all Lipschitz domains and certain fractal domains are John domains.

As a consequence of the Poincaré inequality, the following Korn inequality holds; see [14, Theorem 2.1].

Proposition 2.2. *Let Ω be a domain in $\mathbb{R}^n$, $n \geq 2$. Let $p > 1$ and $a \geq 0$. Suppose the Poincaré inequality $(P_{p,a,a+p})$ holds. Then there exists $C = C(p, a, \Omega)$ such that for every $\mathbf{v} \in W^{1,p}(\Omega, \rho^a)$ satisfying $\int_\Omega \kappa_{i,j}(\mathbf{v})\rho^a\, dx = 0$ for all $1 \leq i, j, \leq n$, the following inequality holds*

$$(K_{p,a}) \qquad \int_\Omega |D\mathbf{v}(x)|^p \rho(x)^a\, dx \leq C \int_\Omega |\epsilon(\mathbf{v})(x)|^p \rho(x)^a\, dx.$$

Remark 2.3. Notice that the Korn inequality from [14, Theorem 2.1] is in a different form, but the same proof there will give the Korn inequality $(K_{p,a})$ as above; see also [13, Section 2].

On each John domain Ω, by [5], it was known that the Poincaré inequality $(P_{p,a,p+a})$ holds for any $p \in [1, \infty)$ and $a \geq 0$.

Corollary 2.4. *Let Ω be a John domain in $\mathbb{R}^n$, $n \geq 2$. Then for any $p > 1$ and $a \geq 0$, there exists $C = C(p, a, \Omega)$ such that for every $\mathbf{v} \in W^{1,p}(\Omega, \rho^a)$ satisfying $\int_\Omega \kappa_{i,j}(\mathbf{v})\rho^a\, dx = 0$ for all $1 \leq i, j \leq n$, the following inequality holds*

$$(K_{p,a}) \qquad \int_\Omega |D\mathbf{v}(x)|^p \rho(x)^a\, dx \leq C \int_\Omega |\epsilon(\mathbf{v})(x)|^p \rho(x)^a\, dx.$$

Diening, Ružička and Schumacher [7] obtained solvability of the divergence equation with A_q weight. Very recently, López García [17, Theorem 1.1] obtained a sharper weighted Korn inequality on John domains.

3 Weighted Korn inequality implies John

In this section, we prove our main result, namely, the weighted Korn inequality also implies John condition, if the domain additionally satisfies the separation property.

The separation property was introduced by Buckley and Koskela [4].

Definition 3.1. (Separation property). A domain $\Omega \subset \mathbb{R}^n$ with a distinguished point x_0 has the separation property if there is a constant $C_s \geq 1$ such that the following holds : for each $x \in \Omega$ there is a curve $\gamma : [0,1] \to \Omega$ with $\gamma(0) = x, \gamma(1) = x_0$, and such that for each t either $\gamma([0,t]) \subset B := B(\gamma(t), C_s\rho(\gamma(t)))$ or for each $y \in \gamma([0,t]) \setminus B$ belongs to a different component of $\Omega \setminus \partial B$ than x_0. In the later case, we call B a separating ball, and call the union of components of $\Omega \setminus B$ not containing x_0 as *B-end* and denoted by E_B.

Every domain Ω, which is quasiconformally equivialent to a uniform domain has the separation property (see [4]). In fact, each finitely connected plane domain has the separation property; see [13, Section 6].

The following result was proved in [3].

Lemma 3.2. *Let $1 \leq b < \infty$ and let x_j be a sequence of nonnegative numbers such that for all $k \in \mathbb{N}$*

$$\Sigma_{j=k}^{\infty} x_j \leq b x_k.$$

Then for every $\alpha \in (0,1]$there exists a constant $c \geq 1$, depending only b, α, such that for all $k \in \mathbb{N}$

$$\Sigma_{j=k}^{\infty} x_j^{\alpha} \leq c x_k^{\alpha}.$$

We also need the well-known Whitney decomposition.

Lemma 3.3. *For any $\Omega \subset \mathbb{R}^n$ there exists a collection W= $\{Q_j\}_{j\in\mathbb{N}}$ of countably many closed dyadic cubes such that*

(i) $\Omega = \cup_{j\in\mathbb{N}} Q_j$, *and the cubes have disjoint interiors,* $(Q_j)^\circ \cap (Q_k)^\circ = \emptyset$,
(ii)$\sqrt{n}\ell(Q_k) \leq dist(Q_k, \partial\Omega) \leq 4\sqrt{n}\ell(Q_k)$ *and*
(iii)$\frac{1}{4}\ell(Q_k) \leq \ell(Q_j) \leq 4\ell(Q_k)$ *whenever* $Q_j \cap Q_k \neq \emptyset$.

Notice that Whitney decompositions of a domain might not be unique, in what follows we fix a decomposition in the proof.

The following result was proved in [13, Proposition 3.3].

Proposition 3.4. *Let $\Omega \subset \mathbb{R}^n$ be a domain satisfying the separation property with constant $C_s \geq 1$ and a distinguished point x_0. For each point $x \in \Omega$, there is a curve γ connecting x to x_0 that satisfies the separation property with constant $5C_s$, and for each Whitney cube $Q \in W$, the set $Q \cap \gamma$ has at most one component.*

We can now provide the last step towards our result.

Theorem 3.5. *Let Ω be a bounded domain of $\mathbb{R}^n$, $n \geq 2$. Assume that Ω satisfies the separation property. Then if there exist $p \in (1,\infty)$ and $a \in [0,\infty)$, such that for all $\mathbf{u} = (u_1, \cdots, u_n) \in W^{1,p}(\Omega, \rho^a)$ satisfying $\int_\Omega (\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i})\rho^a \, dx = 0, 1 \leq i, j \leq n$, it holds that*

$$(K_{p,a}) \qquad \|D\mathbf{u}\|_{L^p(\Omega,\rho^a)} \leq C_K \|\epsilon(\mathbf{u})\|_{L^p(\Omega,\rho^a)},$$

Ω is a John domain.

Proof. Suppose that Ω satisfies the separation property w.r.t. $x_0 \in \Omega$. Let Q_0 be the Whitney cube such that $x_0 \in Q_0 \subset\subset \Omega$.

Let $x \in \Omega$ be a point and let $\gamma : [0,1] \to \Omega$ with $\gamma(0) = x, \gamma(1) = x_0$ be a curve given by the separation condition. Consider the collection W_γ of all Whitney squares that intersect γ. By Proposition 3.4, we may assume that for each Whitney cube Q in W_γ, there is only on component in $Q \cap \gamma$. Therefore, we can order them so that they form a chain $\{Q_j\}_{j=0}^{W_\gamma}$ with $x_0 \in Q_0$ and $W_\gamma \in \mathbb{N}$ depending on x.

By the work of Martio and Sarvas [18], to show that Ω is an John domain, it suffices to show that there exists an absolute constant $C > 0$, independent of the starting point x such that for each $t \in [0,1]$, its has $\rho(\gamma(t)) \geq C \mathrm{diam}(\gamma([0,t]))$.

Claim: For any $z = \rho(\gamma(t))$ with $t \in [0,1]$, let $r := C_s \rho(z)$, and $Q_k \in W_\gamma$ such that $z \in Q_k$. There exists $C > 0$, independent of x, z, such that

$$\sum_{j \geq k} |Q_j|^{\frac{n+a}{n}} \leq C |Q_k|^{\frac{n+a}{n}}. \tag{3.1}$$

Assuming the Claim is proved, then Lemma 3.2 implies that

$$\sum_{j \geq k} \ell(Q_j) \leq C \ell(Q_k),$$

which further implies that

$$\mathrm{diam}(\gamma([0,t])) \leq \sqrt{n} \sum_{j \geq k} \ell(Q_j) \leq C(n) \ell(Q_k) \leq C(n) \rho(\gamma(t)).$$

This guarantees that Ω is a John domain.

Thus we only need to prove the claim (3.1).

Let $B := B(z, r)$, where $r = C_s \rho(z)$ and $z = \gamma(t), t \in [0,1]$, and $Q_k \in W_\gamma$ be such that $z \in Q_k$.

If $B \cap Q_0 \neq \emptyset$, then one has

$$5 C_s \ell(Q_k) \geq C_s \rho(z)/\sqrt{n} \geq \ell(Q_0)/2,$$

and hence,

$$\sum_{j \geq k} |Q_j|^{\frac{n+a}{n}} \leq C(n) \int_\Omega \rho(x)^a \, dx \leq C(n) |Q_k|^{\frac{n+a}{n}}. \tag{3.2}$$

Suppose now that $B \cap Q_0 = \emptyset$. If $\gamma([0,t]) \subset B$ then it holds automatically that

$$\sum_{j \geq k} |Q_j|^{\frac{n+a}{n}} \leq C(n) \int_{\cup_{j \geq k} Q_j} \rho(x)^a \, dx \leq \int_{2B} \rho(x)^a \, dx \leq C(p, a, \Omega) |Q_k|^{\frac{n+a}{n}}. \tag{3.3}$$

Otherwise, let E_B be the B-end. If

$$|B| \geq \left(\frac{\pi^{n/2}}{\Gamma(1 + \frac{n}{2})} \right)^{\frac{a}{n+a}} \frac{|Q_0|}{C_K^{\frac{np}{n+a}} 3^{2p} 2^{2n}},$$

then once more it holds

$$5C_s\ell(Q_k) \geq C_s\rho(z)/\sqrt{n} \geq \ell(Q_0)/2,$$

and similar to (3.2) one has

$$\sum_{j\geq k} |Q_j|^{\frac{n+a}{n}} \leq C(n)|Q_k|^{\frac{n+a}{n}}. \tag{3.4}$$

Suppose now

$$|B| < \left(\frac{\pi^{n/2}}{\Gamma(1+\frac{n}{2})}\right)^{\frac{a}{n+a}} \frac{|Q_0|}{C_K^{\frac{np}{n+a}} 3^{2p}2^{2n}}. \tag{3.5}$$

If $E_B \subset 4B$ then the conclusion is obvious. Otherwise, set

$$\phi(x) := \begin{cases} 0, & \forall x \in \Omega \setminus E_B; \\ 1, & \forall x \in E_B \setminus 2B; \\ \frac{d(x,B)}{r}, & \forall x \in E_B \cap (2B \setminus B). \end{cases} \tag{3.6}$$

then ϕ is a Lipschitz function that vanishes on B.

For each $x = (x_1, \cdots, x_n) \in \Omega$, let $\mathbf{v} = (v_1, v_2, 0, \cdots, 0)$ with

$$\begin{cases} v_1(x_1, \cdots, x_n) = (x_2 - z_2)\phi(x_1, \cdots, x_n), \\ v_2(x_1, \cdots, x_n) = (z_1 - x_1)\phi(x_1, \cdots, x_n), \end{cases} \tag{3.7}$$

where $z = (z_1, \cdots, z_n)$ is the center of B. Then for each $x = (x_1, \cdots, x_n) \in E_B \setminus 2B$,

$$D\mathbf{v}(x) = \begin{pmatrix} 0 & 1\, 0 \cdots 0 \\ -1 & 0\, 0 \cdots 0 \\ 0 & 0\, 0 \cdots 0 \\ \cdots & \\ 0 & 0\, 0 \cdots 0 \end{pmatrix}; \tag{3.8}$$

$D\mathbf{v}(x) = 0$ for all $x \in \Omega \setminus E_B$ and

$$|D\mathbf{v}(x)| \leq 2r|\nabla\phi(x)| + \phi(x) \leq 3$$

for all $x \in E_B \cap (2B \setminus B)$. Choose a vector field $\mathbf{w}$ on Ω as

$$\mathbf{w}(x) = w(x_1, \cdots, x_n) = (-\widetilde{C}x_2, \widetilde{C}x_1, 0, \cdots, 0),$$

where $\widetilde{C}$ satisfies

$$2\widetilde{C}\int_\Omega \rho(x)^a dx = \int_\Omega \left(\frac{\partial v_1}{\partial x_2} - \frac{\partial v_2}{\partial x_1}\right)\rho(x)^a dx,$$

Now set $\mathbf{u} = \mathbf{v} + \mathbf{w}$. One has that $\mathbf{u} = (u_1, u_2, \cdots)$, where $u_i = v_i + w_i, i = 1, 2$, is Lipchitz continuous on Ω and satisfies

$$\int_\Omega \left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right)\rho(x)^a dx = \int_\Omega \left(\frac{\partial v_1}{\partial x_2} - \frac{\partial v_2}{\partial x_1}\right)\rho(x)^a dx - 2\widetilde{C}\int_\Omega \rho(x)^a dx = 0.$$

Applying the weighted versions of the Korn inequality $(K_{p,a})$ to $\mathbf{u}$, and noticing that $\epsilon(\mathbf{w}) \equiv 0$, we obtain

$$
\begin{aligned}
\|D\mathbf{u}\|_{L^p(\Omega,\rho^a)} &\le C_K \|\epsilon(\mathbf{u})\|_{L^p(\Omega,\rho^a)} = C_K \|\epsilon(\mathbf{v})\|_{L^p(\Omega,\rho^a)} \\
&= C_K \left(\int_{E_B \cap 2B} |\epsilon(\mathbf{v})|^p \rho(x)^a \, dx \right)^{1/p} \le 3C_K \left(\int_{E_B \cap 2B} \rho(x)^a \, dx \right)^{1/p} \\
&\le 3C_K r^{a/p} |E_B \cap 2B|^{1/p}.
\end{aligned} \tag{3.9}
$$

By the construction of $\mathbf{v}$ we have $\mathbf{v} = 0$ on $\Omega \setminus E_B$, and therefore by (3.5),

$$
\begin{aligned}
|\widetilde{C}| \left(\int_{Q_0} \rho(x)^a \, dx \right)^{1/p} &\le \left(\int_{\Omega \setminus E_B} |Du|^p \rho(x)^a \, dx \right)^{1/p} \\
&\le 3C_K (2C_s \rho(z))^{a/p} |E_B \cap 2B|^{1/p} \\
&\le 3C_K (2C_s \rho(z))^{a/p} |2B|^{1/p} \\
&\le 3C_K 2^{(2a+n)/p} r^{a/p} |B(z,r)|^{1/p} \\
&= 3C_K 2^{(2a+n)/p} \left(\frac{\Gamma(1+\frac{n}{2})}{\pi^{n/2}} \right)^{a/np} |B(z,r)|^{(n+a)/np} \\
&\overset{(3.5)}{\le} \frac{1}{3} |Q_0|^{(n+a)/np}.
\end{aligned} \tag{3.10}
$$

On the other hand, one has

$$
\left(\int_{Q_0} \rho(x)^a \, dx \right)^{1/p} \ge (\sqrt{n}\ell(Q_0))^{a/p} |Q_0|^{1/p} \ge |Q_0|^{(n+a)/np}.
$$

From this, we see that $|\widetilde{C}| \le 1/3$. This together with (3.9) implies that

$$
\begin{aligned}
C \left(\int_{E_B \setminus 2B} \rho(x)^a \, dx \right)^{1/p} &\le \|D\mathbf{u}\|_{L^p(\Omega,\rho^a)} \le C_K \|\epsilon(\mathbf{u})\|_{L^p(\Omega,\rho^a)} = C_K \|\epsilon(\mathbf{v})\|_{L^p(\Omega,\rho^a)} \\
&= C_K \left(\int_{E_B \cap 2B} |\epsilon(\mathbf{v})|^p \rho(x)^a \, dx \right)^{1/p} \\
&\le 3C_K \left(\int_{E_B \cap 2B} \rho(x)^a \, dx \right)^{1/p} \\
&\le C \left(\int_{Q \in W:\, Q \cap \gamma(t) \neq \emptyset} \rho(x)^a \, dx \right)^{1/p}. \\
&\le C |Q_k|^{\frac{n+a}{np}}.
\end{aligned}
$$

This together with the separation property then implies that

$$
\sum_{j \ge k} |Q_j|^{\frac{n+a}{n}} \le C(n) \int_{E_B \cup B} \rho(x)^a \, dx \le C(n) |Q_k|^{\frac{n+a}{n}}. \tag{3.11}
$$

The estimates (3.2), (3.3), (3.4), (3.5) and (3.11) together imply (3.1), which completes the proof of the theorem. □

We can now finish the proof of the main result.

Proof of Theorem 1.1. $(i) \to (ii)$ was contained in Proposition 2.2, while $(ii) \to (i)$ was contained in Theorem 3.5.

Finally, (iii) implies (ii) obviously. On the other hand, (ii) implies (i), i.e., Ω is John, which implies (iii) by Corollary 2.4. □

Acknowledgments

R. Jiang was partially supported by NNSF of China (11671039).

References

[1] G. Acosta, R.G. Durán, A.L. Lombardi, Weighted Poincaré and Korn inequalities for Hölder α domains, Math. Methods Appl. Sci. 29 (2006) 387-400.

[2] G. Acosta, R.G. Durán, M.A. Muschietti, Solutions of the divergence operator on John domains, Adv.Math. 206 (2006) 373-401.

[3] K. Astala, F.W. Gehring, Quasiconformal analogues of theorems of theorems of Koebe and Hardy-Littlewood, Michigan Math. J. 32 (1985) no. 1, 99-107.

[4] S. Buckley, P. Koskela, Sobolev-Poincaré implies John, Math. Res. Lett. 2 (1995), 577-593.

[5] S.K. Chua, R.L. Wheeden, Self-improveing properties of inequalities of Poincaré type on s-John domains, Pacific J. Math. 250 (2011) 67-108.

[6] A. Cianchi, Korn type inequalities in Orlicz spaces, J. Funct. Anal. 267 (2014) 2313-2352.

[7] L. Diening, M. Ružička, Schumacher K., A decomposition technique for John domains, Ann. Acad. Sci. Fenn. Math., 35 (2010), 87-114.

[8] R.G. Durán, F. López García, Solution of the divergence and Korn inequalities on domains with an external cusp, Ann.Acad. Sci. Fenn. Math. 35 (2010) 421-438.

[9] G. Duvaut, J.-L. Lions, Inequalities in Mechanics and Physics, Springer, 1976.

[10] K.O. Friedrichs, On the doundary-value problems of the thoery of elasticity and Korn's inequality, Ann. of Math. 48 (2) (1947) 441-471.

[11] P. Hajłasz, P. Koskela, Isoperimetric inequalities and imbeding theroems in irregular domains, J. Lond. Math. Soc. (2) 58 (1998) 425-450.

[12] C.O. Horgan, Korn's inequalities and their applications in continuum mechanics, SIAM Rev. 37 (1995) 491-511.

[13] R. Jiang, A. Kauranen, Korn's inequality and John domain, Calc. Var. Partial Differential Equations, to appear (arXiv:1603.01047).

[14] R. Jiang, A. Kauranen, Korn inequality on irregular domains, J. Math. Anal. Appl. 423 (2015) 41-59.

[15] F. John, Rotation and strain. Comm. Pure Appl. Math, 1961, 14:391-413.

[16] V.A. Kondratiev, O.A. Oleinik, On Korn's inequalities, C. R. Acad. Sci. Paris Sér. I Math. 308 (1989) 483-487.

[17] F. López García, Weighted Korn inequality on John domains, arXiv:1612.04445.

[18] O. Martio, J. Sarvas, Injectivity theorems in plane and space, Ann. Acad. Sci. Fenn. Ser. A I Math. 4 (1979), 383-401.

[19] J. Nečas, Les méthodes directes en théorie des équations elliptiques. (French) Masson et Cie (Eds.), Paris; Academia, Editeurs, Prague, 1967.

Manru Jiang[1] & Renjin Jiang[2]

1. School of Mathematical Sciences, Beijing Normal University, Laboratory of Mathematics and Complex Systems, Beijing 100875, China

2. Center for Applied Mathematics, Tianjin University, Tianjin 300072, China

E-mail addresses:
201521130037@mail.bnu.edu.cn
rejiang@tju.edu.cn