One Time-step Particle Smoothing for Radio Range-based Indoor Position Tracking

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In the context of sequential estimation of radio range-based indoor position tracking, Bayesian smoothing framework is promising as involving past, present and future observations. The performance and practicability of a smoothing method greatly depend on how many and how future observations are incorporated. Aiming at Real-time Locating Systems (RTLS), we propose to implement smoothing on Sequential Monte Carlo (SMC) methods, including four popular Bayesian smoothing methods and a novel one time-step Smoothed Filtering (SF) algorithm. The smoothing algorithms are evaluated through Two Dimensional (2D) position tracking on a real-world indoor test-bed. We present results that the proposed SF improves tracking performance requiring very limited computation and memory, which is applicable for real-time indoor position tracking. Moreover, the one time-step smoothing form is validated to mitigate ranging errors and smooth positioning trajectories.

Introduction: Real-time and continuous positioning of wireless systems is the key issue of indoor location-aware service, emergency response and robotics, etc. However, in radio range-based positioning, either the ranging that sensors measure or the motion of a target is usually difficult to model accurately. Bayesian smoothing methods are promising to combat imprecise and scarce measurement problems, as involving not only the past and present observations ($\mathbf{z}_{1:t}$) at the current time (t) but also the future ones ($\mathbf{z}_{t:T}$, $t<T$) a few time ahead.

Research have proposed a class of smoothing algorithms in a recursive Bayesian framework, i.e., the Gaussian Rauch-Tung-Striebel (RTS) smoother [1], the Forward Filtering Backward Smoothing (FFBSm) [2] and the Two-Filter Smoother (TFS) [3, 4]. However, they are either computation costly or require to predefine samples. Alternative Monte Carlo methods, Sequential Monte Carlo (SMC, also known as particle filter) methods [6], provide a particle-based state propagation [7, 5].

This paper focuses on a real-time state smoothing based on the observations up to one time-step after the present, defined as *one time-step smoothing* ($p(\mathbf{x}_t|\mathbf{z}_{1:t+1})$). In order to improve the forward particle propagation, we propose a smoothed filtering algorithm in a SMC method (Generic Particle Filter (GPF)), namely smoothed filtering (SF). Instead of deriving the posterior from the prediction density, SF recursively propagates the posterior from the smoothing density. We also implement four popular smoothing solutions in GPF: Forward Filtering Backward Smoothing (FFBSm), Forward Filtering Backward Simulation (FFBSi), Two-filter Smoothing (TFS) and Fast Two-filter Smoothing (TFS_{fast}). The aforementioned smoothing algorithms are evaluated over our indoor tracking test-bed with Time-of-Flight (TOF) ranging. Experimental results validate the improvements in accuracy and smoothness of the one-time step smoothing framework on real-world position tracking.

One time-step smoothing: Indoor radio range-based positioning system observes severe measurement noise or failures [8], due to system noise, multi-path effect, Non-Line-Of-Sight (NLOS) propagations, unknown wireless interference, etc. From a Bayesian perspective of sequential position estimation, filtering represents the posterior ($p(\mathbf{x}_t|\mathbf{z}_{1:t})$) of the state given the observations up to the current time; smoothing corresponds to the density ($p(\mathbf{x}_t|\mathbf{z}_{1:T})$) based on the observations up to some later time (T, $t<T$). To recur the state recursion, it essentially applies a Hidden Markov Model (HMM) of order one.

The smoothing methods can obviously provide better approximations of the state probability if the future observations are enough. However, the smoothing recursion ($p(\mathbf{x}_t|\mathbf{z}_{1:T})$) involves the observations many time-steps ahead ($t \ll T$), it can be computation, storage, and time consuming. To deal with the problem, it is preferable to form the smoothing density from a few time-steps of observations. Aiming at real-time tracking, we propose the smoothing density of the one time-step SMC with

$$T = t+1 \ . \tag{1}$$

The *one time-step smoothing*, that compute the sequence of conditional density, is defined as

$$p(\mathbf{x}_t|\mathbf{z}_{1:t+1}) \ . \tag{2}$$

Current methods of particle smoothing

We apply four popular Bayesian smoothing algorithms of one time-step recursion in GPF as following.

Forward Filtering Backward Smoothing (FFBSm)

The smoothing density of FFBSm is deduced from a forward-backward expression, with the one time-step form as

$$p(\mathbf{x}_t|\mathbf{z}_{1:t+1}) = p(\mathbf{x}_t|\mathbf{z}_{1:t}) \int \frac{p(\mathbf{x}_{t+1}|\mathbf{z}_{1:t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)}{\int p(\mathbf{x}_{t+1}|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{z}_{1:t})\, d\mathbf{x}_t}\, d\mathbf{x}_{t+1} \ . \tag{3}$$

The filtering density ($p(\mathbf{x}_{t+1}|\mathbf{z}_{1:t+1})$) in (3) can be computed by your favorite forward filter, as we use the GPF method [9].

Forward Filtering Backward Simulation (FFBSi)

In order to remedy the high computation of FFBSm in (3), FFBSi [10] defines the smoothing density as

$$p(\mathbf{x}_t|\mathbf{x}_{t+1}, \mathbf{z}_{1:t}) = \frac{p(\mathbf{x}_t|\mathbf{z}_{1:t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)}{p(\mathbf{x}_{t+1}|\mathbf{z}_{1:t})} \ . \tag{4}$$

Instead re-weighting particles as FFBSm, FFBSi samples from the backward smoothing density

$$\tilde{\mathbf{x}}_t \sim p(\mathbf{x}_t|\mathbf{z}_{t+1}) \ . \tag{5}$$

Two Filter Smoothing (TFS)

TFS [11] is a well-established alternative to FFBSm, which obtains the smoothing density from two independent filters (the forward and the backward filters). The one time-step TFS is formulated as

$$p(\mathbf{x}_t|\mathbf{z}_{1:t+1}) \propto \underbrace{p(\mathbf{x}_t|\mathbf{z}_{1:t})}_{\text{Forward filter}} \underbrace{\int p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)\, d\mathbf{x}_{t+1}}_{\text{Backward filter}} \ . \tag{6}$$

Fast Two-filter Smoothing (TFS_{fast})

Differing from the conventional TFS, TFS_{fast} [12] draws new particles from the empirical density and approximates the smoothing density by

$$\begin{aligned} p(\mathbf{x}_t|\mathbf{z}_{1:t+1}) &\propto p(\mathbf{x}_t|\mathbf{z}_{1:t})p(\mathbf{z}_{t+1}|\mathbf{x}_t) \\ &\propto p(\mathbf{x}_t|\mathbf{z}_{1:t}) \int \frac{p(\tilde{\mathbf{x}}_{t+1}|\mathbf{z}_{t+1})p(\tilde{\mathbf{x}}_{t+1}|\mathbf{x}_t)}{\lambda_{t+1}(\tilde{\mathbf{x}}_{t+1})}\, d\tilde{\mathbf{x}}_{t+1} \ . \end{aligned} \tag{7}$$

with $\lambda_{t+1}(\mathbf{x}_{t+1})$ being the artificial prior.

Proposed Smoothed Filtering (SF)

The aforementioned smoothing methods formulate the smoothing density ($p(\mathbf{x}_t|\mathbf{z}_{1:t+1})$) from the current ($p(\mathbf{x}_t|\mathbf{z}_{1:t})$) and future ($p(\mathbf{x}_{t+1}|\mathbf{z}_{1:t+1})$) posterior. The shortcoming is that the smoothing density influences only the backward density rather than the forward probability propagation.

Since FFBSm and TFS have not incorporated the smoothing density into the state recursion, we propose to propagate the posterior from the smoothing density, namely, Smoothed Filtering (SF) as

$$p(\mathbf{x}_{t+1}|\mathbf{z}_{1:t+1}) = \int p(\mathbf{x}_t|\mathbf{z}_{1:t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{z}_{1:t+1})\, d\mathbf{x}_t \ . \tag{8}$$

Form a Markov process of order one, one gets that

$$\begin{aligned} p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{z}_{1:t+1}) &\overset{\text{Markov}}{=} p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{z}_{t+1}) \\ &= \frac{p(\mathbf{x}_{t+1}, \mathbf{z}_{t+1}|\mathbf{x}_t)}{p(\mathbf{z}_{t+1}|\mathbf{x}_t)} \\ &= \frac{p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1}\mathbf{x}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t)}{p(\mathbf{z}_{t+1}|\mathbf{x}_t)} \\ &= \frac{p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)}{p(\mathbf{z}_{t+1}|\mathbf{x}_t)} \\ &= \frac{p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)}{\int p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1})p(\mathbf{x}_{t+1}|\mathbf{x}_t)\, d\mathbf{x}_{t+1}} \ . \end{aligned} \tag{9}$$

The factor $p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{z}_{1:t+1})$ can be derived by (9). In the condition of the low velocity of our robot (averagely 0.5 m/s), we take the approximation $p(\mathbf{z}_{t+1}|\mathbf{x}_{t+1}) \approx p(\mathbf{z}_{t+1}|\mathbf{x}_t)$ leading to

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{z}_{1:t+1}) \approx p(\mathbf{x}_{t+1}|\mathbf{x}_t) \ . \tag{10}$$

Similar to the TFS, the smoothing density of SF is

$$p(\mathbf{x}_t|\mathbf{z}_{1:t+1}) \propto p(\mathbf{x}_t|\mathbf{z}_{1:t})p(\mathbf{z}_{t+1}|\mathbf{x}_t) \ . \tag{11}$$

Results and analysis: The aforementioned smoothing algorithms are implemented in an indoor tracking test-bed as introduced in the work [13], which consists of a robot and a network of Nanotron NanoPAN sensors with TOF ranging. The experiment is carried out in a typical indoor scenarios, the halls and classrooms with a mobile trajectory over 50 meters.

Quantitative results

The competing algorithms are performed on the one-step smoothing frame, taking the same initialization, particle size of GPF ($N_{\mathrm{p}} = 49$), Gaussian measurement model, Gaussian random motion model and residual resampling strategy. The quantitative results of 6000 positioning executions in the experiment are listed in Table .

Table 1: Comparison of the one time-step smoothing on SMC, with the positioning results of the indoor experiment

Algorithms	$\mathrm{MEAN_p}$/meter	$\mathrm{RMSE_p}$/meter	$\mathrm{MAX_p}$/meter	Runtime/s
GPF	1.62	1.88	5.71	22
FFBSm	1.58	1.84	5.69	365
FFBSi	1.62	1.89	6.22	29
TFS	1.52	1.75	5.87	751
$\mathrm{TFS_{fast}}$	1.61	1.86	5.95	114
SF	1.47	1.67	4.70	30

Table demonstrates that FFBSi and $\mathrm{TFS_{fast}}$ almost make no improvements on GPF. We consider that the empirical sampling of FFBSi and $\mathrm{TFS_{fast}}$ might introduce extra variance that cancels out the smoothing effect. The FFBSm and TFS obtain small improvements. It is explained that FFBSm and TFS only influences the backward density rather than the posterior; thus, the sample divergence of the forward probability recursion remains unaltered. The proposed SF observes the lowest values of the $\mathrm{MEAN_p}$, $\mathrm{RMSE_p}$, and $\mathrm{MAX_p}$; this is a consequence that the probability propagation is derived from the smoothing density instead of the prediction density. Furthermore, the complexity of the above smoothing algorithms is $\mathrm{O}(N_{\mathrm{p}})^2$, while the runtime indicates the efficiency of SF. Therefore, the one time-step smoothing form can be effective if the smoothing density can be used to refine the forward state propagation.

Positioning behavior

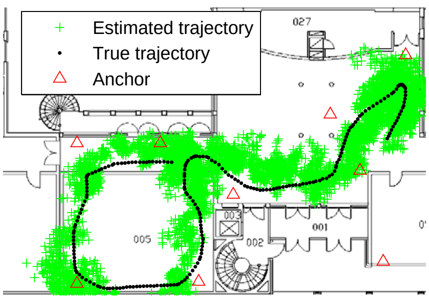

Fig. 1 *Positioning behavior of SF (one time-step) over a mobile trajectory on the building floor plan with classrooms and halls: The connected dots denotes the ground truth of the mobile trajectory; the scatter plot '+' is the estimations; '△' for the anchors.*

Figure 1 depicts the estimated trajectory of the smoothing algorithms on the floor plan, which indicates that SF achieves a smoothness behavior. It causes large divergence to the true trajectory at some test sites, where are the NLOS scenarios in our indoor environment. The results present that the proposed one time-step smoothing SF is applicable for real-time indoor position tracking.

Conclusion: To deal with nonlinearity and non-Gaussianity of indoor range-based position tracking, five smoothing algorithms are applied to the SMC implementation. Aware of the real-time constraint, we focus on the smoothing frame in one time-step form. By validation in a real-world indoor tracking experiment, it is summarized that the one time-step smoothing is of high relevance for both reducing the state uncertainty and smoothing the representation in real-time tracking. The FFBSm, TFS and their variations are not effective in one time-step recursion, by reason that the smoothing density is not propagated into the forward state propagation. The SF achieves notable improvements, as deriving the posterior from the smoothing density. Since SF requires no other assumptions, offline training or high complexity, it is practical for indoor range-based tracking.

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References

1 Kazufumi Ito and Kaiqi Xiong.
Gaussian filters for nonlinear filtering problems.
Automatic Control, IEEE Transactions on, 45(5):910–927, 2000.

2 Genshiro Kitagawa.
Non-gaussian state-space modeling of nonstationary time series.
Journal of the American statistical association, 82(400):1032–1041, 1987.

3 D Fraser and J Potter.
The optimum linear smoother as a combination of two optimum linear filters.
Automatic Control, IEEE Transactions on, 14(4):387–390, 1969.

4 Y. Zhuang, Y. Li, L. Qi, H. Lan, J. Yang, and N. El-Sheimy.
A two-filter integration of mems sensors and wifi fingerprinting for indoor positioning.
IEEE Sensors Journal, 16(13):5125–5126, July 2016.

5 S. Nagappa, E. D. Delande, D. E. Clark, and J. Houssineau.
A tractable forwardÍc backward cphd smoother.
IEEE Transactions on Aerospace and Electronic Systems, 53(1):201–217, Feb 2017.

6 Johan Westerborn and Jimmy Olsson.
Efficient particle-based online smoothing in general hidden markov models.
In *Acoustics, Speech and Signal Processing (ICASSP), 2014 IEEE International Conference on*, pages 8003–8007. IEEE, 2014.

7 Fredrik Lindsten and Thomas B Schön.
Backward simulation methods for monte carlo statistical inference.
Foundations and Trends in Machine Learning, 6(1):1–143, 2013.

8 N.A. Alsindi, B. Alavi, and K. Pahlavan.
Measurement and modeling of ultrawideband toa-based ranging in indoor multipath environments.
Vehicular Technology, IEEE Transactions on, 58(3):1046–1058, 2009.

9 M Sanjeev Arulampalam, Simon Maskell, Neil Gordon, and Tim Clapp.
A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking.
IEEE Transactions on Signal Processing, 50(2):174–188, 2002.

10 Simon J Godsill, Arnaud Doucet, and Mike West.
Monte carlo smoothing for nonlinear time series.
Journal of the american statistical association, 99(465), 2004.

11 Drew Creal.
A survey of sequential monte carlo methods for economics and finance.
Econometric Reviews, 31(3):245–296, 2012.

12 Paul Fearnhead, David Wyncoll, and Jonathan Tawn.
A sequential smoothing algorithm with linear computational cost.
Biometrika, 97(2):447–464, 2010.

13 Simon Schmitt, Heiko Will, Benjamin Aschenbrenner, Thomas Hillebrandt, and Marcel Kyas.
A reference system for indoor localization testbeds.
In *Indoor Positioning and Indoor Navigation (IPIN), International Conference on*, pages 1–8. IEEE, 2012.